

**SACRED HEART COLLEGE (AUTONOMOUS)**

**Department of Mathematics**

**M.Sc. Mathematics**

**Course plan**

**Academic Year 2015 – 16**

**Semester 4**

## COURSE PLAN

### COMBINATORICS

**TEXT BOOK : CHEN CHUAN-CHONG ,KOH KHEE MENG,PRINCIPLES AND TECHNIQUES IN COMBINATORICS,WORLD SCIENTIFIC,1999.**

#### COURSE OBJECTIVES

To understand permutations and combinations & its applications.

To introduce Pigeonhole principle and Ramsey numbers and its applications.

To understand generating functions and its applications.

To introduce recurrence relation and methods to solve that.

Sessions	Topic	Method	Remarks/Reference
1	Two basic counting principles	Lecture,Group discussion	
2	Problems	Group Discussion,Problem solving	
3	Problems, Permutations	Group Discussion,Problem solving	
4	Problems	Group Discussion,Problem solving	
5	Circular permutations and Principle of complementation and problems	Lecture,Group Discussion,Problem solving	
6	Problems, Combinations	Lecture,Group Discussion,Problem solving	
7	Problems , $S(n, r)$	Lecture,Group Discussion,Problem solving	

8	Problems , Injection and bijection principle	Lecture,Group Discussion,Problem solving	
9	Problems	Group Discussion,Problem solving	
10	Arrangemrnts and problems	Lecture,Group Discussion,Problem solving	
11	Selection with repetitions	Lecture,Group Discussion,Problem solving	
12	Problems	Lecture,Group Discussion,Problem solving	
13	Distribution problems-different cases	Lecture,Group Discussion,Problem solving	
14	Problems	Group Discussion,Problem solving	
15	Extra problems from exercise	Group Discussion,Problem solving	
16	Extra problems from exercise	Group Discussion,Problem solving	
17	Introduction to binomial coefficients	Lecture,Group Discussion	
18	Binomial theorem and proof	Lecture,Group Discussion,	
19	Combinatorial identities	Lecture,Group Discussion,	
20	More identities	Lecture,Group Discussion,Problem solving	
21	Test of Module 1	1 hr	

22	The Pascal's triangle	Lecture,Group Discussion,Problem solving	
23	Problems	Group Discussion,Problem solving	
24	Introduction to pigeonhole principle	Lecture,Group Discussion,Problem solving	
25	Basic problems based on PP	Lecture,Group Discussion,Problem solving	
26	More examples on PP	Group Discussion,Problem solving	
27	Problems	Lecture,Group Discussion,Problem solving	
28	Ramsey type problems	Group Discussion,Problem solving	
29	More problems and Ramsey numbers	Lecture,Group Discussion,Problem solving	
30	Problems, Generalised Pigeonhole Principle	Lecture,Group Discussion,Problem solving	
31	Theorems on Bounds for Ramsey numbers	Lecture,Group Discussion,Problem solving	
32	Problems	Group Discussion,Problem solving	
33	Extra problems from exercise	Group Discussion,Problem solving	
34	CIA-1	1 HR	

35	Basic Inclusion and Exclusion principle	Lecture,Group Discussion,Problem solving	
36	Principle of inclusion and exclusion(PIE) and its proof	Lecture,Group Discussion	
37	Problems	Group Discussion,Problem solving	
38	Generalisation of PIE and problems	Lecture,Group Discussion,Problem solving	
39	Generalised Principle of inclusion and exclusion(GPIE) and proof	Lecture,Group Discussion	
40	Problems	Group Discussion,Problem solving	
41	Extra problems from exercise	Group Discussion,Problem solving	
42	Shortest route problem	Group Discussion,Problem solving	
43	Integer solutions for an equation	Group Discussion,Problem solving	
44	More problems	Group Discussion,Problem solving	
45	Surjective mappings	Lecture,Group Discussion,Problem solving	
46	Sterling numbers of second kind	Lecture,Group Discussion,Problem solving	

47	Derangements and problems	Lecture,Group Discussion,Problem solving	
48	Generalisation of Derangements	Lecture,Group Discussion,Problem solving	
49	Problems	Group Discussion,Problem solving	
50	Test	1 hr	
51	Ordinary generating functions ,Cauchy product	Lecture,Group Discussion,	
52	Examples	Lecture,Group Discussion,Problem solving	
53	Generating functions for different sequences	Lecture,Group Discussion,Problem solving	
54	Problems	Group Discussion,Problem solving	
55	Some modeling problems	Lecture,Group Discussion,Problem solving	
56	Examples	Group Discussion,Problem solving	
57	Partitions of integers	Lecture,Group Discussion,Problem solving	
58	Theorems based on partitions	Lecture,Group Discussion	
59	Ferrers Diagram and problems	Lecture,Group Discussion,Problem solving	

60	Theorems and problems	Lecture,Group Discussion,Problem solving	
61	Exponential generating functions and examples	Lecture,Group Discussion,Problem solving	
62	Exponential generating functions for permutations and examples	Lecture,Group Discussion,Problem solving	
63	More examples	Lecture,Group Discussion,Problem solving	
64	Distribution problems	Lecture,Group Discussion,Problem solving	
65	Problems from exercise	Group Discussion,Problem solving	
66	Introduction to recurrence relations	Lecture,Group Discussion,Problem solving	
67	Tower of Hanoi problem	Lecture,Group Discussion,Problem solving	
68	Finding number of parallelograms in the nth sub-division of an equilateral triangle	Lecture,Group Discussion,Problem solving	
69	Method to solve Linear homogenous recurrence relations	Lecture,Group Discussion,Problem solving	
70	Problems	Group Discussion,Problem solving	
71	More problems	Group Discussion,Problem solving	

72	Method to solve General Linear recurrence relations	Lecture,Group Discussion,Problem solving	
73	Problems	Group Discussion,Problem solving	
74	Number of ways of colouring a circle with n sectors using k colours with certain conditions	Lecture,Group Discussion,Problem solving	
75	Finding the determinant of a special type matrix using recurrence relation	Lecture,Group Discussion,Problem solving	
76	Extra problems	Group Discussion,Problem solving	
77	CIA -2	2 hrs	
78	Revision		
79	Revision		



## COURSE PLAN

### OPERATION RESEARCH

**COURSE OBJECTIVES** To introduce the concept of inventory control, Non-linear programming problem, Dynamic programming, Integer linear programming problem.

**Text - 1-** K.V. Mital and C. Mohan, **Optimization Methods in Operation Research and Systems Analysis, 3rd edition.**

**Text -2-** Ravindran, Philips and Solberg. **Operations Research Principle and Practice, 2nd edition, John Wiley and Sons.**

**Text - 3 -**Man Mohan, P.K. Gupta and Kanti Swarup, **Operations Research, Sultan Chand and Sons. Module I: INVENTORY MODELS**

Sessions	Topic	Method
1.	Module 1 - Introductory Session	Lecture
2.	Introductory session – Costs associated with inventory	Lecture
3.	Introductory session – Basic notations in inventory control	Lecture
4.	Factors affecting inventory	Lecture,
5.	Economic order quantity	Lecture,
6.	EOQ problems without shortage - equal time period	Lecture,
7.	Characteristics of the model	Lecture,
8.	EOQ problems without shortage - unequal time interval	Lecture, Group Discussion,
9.	Characteristics of the model	Lecture,

10.	EOQ problems without shortage - instantaneous production	Lecture, Group Discussion,
11.	Characteristics of the model	Lecture, Group Discussion,
12.	Deterministic inventory Problems with shortages - Fundamental Problem	Problem Solving
13.	Deterministic inventory Problems with shortages - Fixed time period	Lecture, Group Discussion, Problem Solving
14.	Deterministic inventory Problems with shortages - Instantaneous production	Problem Solving
15.	Characteristics of the model	Lecture
16.	EOQ problem with single price - breaks	Problem Solving
17.	EOQ problem with multiple price - breaks	Lecture,
18.	Problems	Group discussion
19.	Problem solving sessions	Lecture,
20.	Problem solving sessions	Lecture,
21.	Module 2 : Introduction	Lecture,
22.	Basic concepts of NLPP	Lecture,

23.	Taylor's series in single variable and two variables	Lecture,
24.	n- dimensional Taylor's series	Lecture,
25.	Fibonacci search method	Lecture,
26.	Problems	Lecture,
27.	Golden section search method	Lecture,
28.	Problems	Lecture,
29.	Hooke and Jeeves search algorithm	Lecture,
30.	Problem	Lecture,
31.	Gradient projection method	Lecture,
32.	Problem	Lecture,
33.	Newton's method	Lecture, Group Discussion, Problem Solving

34.	Problems	Problem Solving
35.	Problems	Lecture, Group Discussion, Problem Solving
36.	Kuhn-tucker conditions	Problem Solving
37.	Problems	Lecture, Group Discussion, Problem Solving
38.	Module 3 : Introduction of DPP	Problem Solving
39.	Minimum path problem	Lecture,
40.	Single additive constraint, additively separable return	Lecture,
41.	problems	Problem Solving
42.	Single multiplicative constraints, additively separable return	Lecture,
43.	Problems	Problem Solving
44.	Single additive constraint, multiplicatively separable return	Lecture,
45.	Problems	Lecture,

46.	Examples of failure	Lecture,
47.	Decomposition – backward and forward recursions	Lecture,
48.	Theorems	Lecture,
49.	Systems with more than one constraint	Lecture,
50.	problems	Lecture,
51.	Problem	Lecture,
52.	Problems	Lecture,
53.	Module 4 - Introduction	Lecture,
54.	I.L.P in two dimensional space	Lecture,
55.	– General I.L.P. and M.I.L.P problems	Lecture,
56.	Theorems	Lecture,
57.	cutting planes - ILP	Lecture,

58.	Problems	Lecture, Group Discussion, Problem Solving
59.	cutting planes - MILP	Lecture, Group Discussion, Problem Solving
60.	Branch and Bound method	Lecture, Group Discussion, Problem Solving
61.	Problems solving	Lecture,
62.	Problems solving	Lecture,
63.	Introduction to Graphs	Lecture,
64.	Definitions and notations	Lecture,
65.	minimum path problem	Lecture,
66.	Problems	Lecture, Group Discussion,
67.	Minimum path problem with arborescence	Lecture,
68.	Problems	Lecture,
69.	Spanning tree of minimum length	Lecture,
70.	Problems	Lecture, Group Discussion,
71.	Problem of minimum potential difference	Lecture,
72.	Problems	Lecture,
73.	scheduling of sequential activities	Lecture,
74.	Maximum flow problem, algorithm	Lecture,

75.	Problems	Lecture,
76.	Theorems	Lecture,
77.	Maximum flow minimum cut theorem	Lecture,
78.	Revision	
79.	Revision	
80.	Revision	
81.	Revision	

References:

1. S.S.Rao, Optimization Theory and Applications, 2<sup>nd</sup> edition, NewAge International Pvt.
2. J.K.Sharma, Operations Research : Theory and Applications, Third Edition, Macmillan IndiaLtd
3. Hamdy A. Taha, Operations Research – An introduction, 6<sup>th</sup> edition, Prentice Hall of India Pvt. Ltd.

**COURSE PLAN**  
**THEORY OF WAVELETS**

**COURSE OBJECTIVES**

\*To study basics of Wavelet theory.

\*To identify **the areas of application of wavelets**

**\*To** study wavelet theory through Linear algebra

\*How the theory of wavelets is linked with data compression , analysis of audio and video signals etc .

Sessions	Topics to be taught	Method adopted	Remark
1	Fundamentals of vector spaces and metric spaces	Seminar , lecture , group work	Bridge course
2	Linear transformations and its matrix representations	Lecture , assignment, group work	Bridge course
3	$l^2(N)$ and $l^2(Z_N)$	Seminar , lecture , assignment	Introduction
4	Introduction of the orthonormal basis $\{ E_1, E_2, E_3, \dots, E_{N-1} \}$	Lecture	More thrust will be given to examples
5	The properties of the above basis	Seminar , lecture	
6	Introduction of the fourier basis F	Lecture , assignment	Examples will be discussed
7	Introduction of discrete fourier transform	Lecture	
8	Matrix representation of the discrete fourier transform	Lecture , assignments	Sufficient number of examples will be discussed
9	Inverse discrete Fourier transform and its matrix representation	Lecture , seminar	More importance will be given for examples



10	Translation by k operator and its Fourier transform	Lecture , group discussion	Examples will be discussed
11	Conjugate of a vector and its Fourier transform	Lecture	Examples will be discussed
12	Translation invariant linear transformation	Lecture , assignment	
13	The result saying that a translation invariant linear transformation is diagonalizable	lecture	
14	Convolution of two vectors in $l^2(Z_N)$	Lecture	Sufficient number of examples will be discussed
15	Convolution operator and the lemma showing that a convolution operator is translation invariant	lecture	A good number of examples will be discussed
16	The Dirac delta function and its properties	Lecture	
17	The DFT of convolution	Lecture , seminar	
18	Spatially localized and frequency localised bases of $l^2(Z_N)$	Assignment , lecture, group discussion	
19	Conjugate reflection and its DFT	lecture	Verification of the results will be done
20	Components of a convolution in terms of inner products	Seminar , lecture	The results will be verified
21	The necessary and sufficient condition for $\{R_k w\}_{k=0}^{N-1}$ to be an orthonormal basis for $l^2(Z_N)$	Lecture	
22	Introduction of first stage wavelet basis for $l^2(Z_N)$	lecture	
23	Introduction of $z^*$ and its DFT	Lecture, assignments	Examples will be discussed
24	The necessary and sufficient condition for $\{R_k w\}_{k=0}^{M-1}$ to be an orthonormal set with M elements	lecture	
25	Introduction of the system matrix $A(n)$ of two vectors u and v	Lecture , seminar	Examples will be discussed

26	The necessary and sufficient condition for two vectors $u$ and $v$ to generate a first stage wavelet basis for $l^2(\mathbb{Z}_N)$	Lecture	Illustrations will be given
27	Some examples of first stage wavelet basis	Assignment , seminar	
28	Description of first Shannon basis and first stage real Shannon basis	lecture	
29	Introduction with sufficient explanations of first stage Haar basis	Lecture	
30	Lemma 3.12	lecture	
31	Up sampling and down sampling operators and their properties	Lecture , seminar	Examples and illustrations will be discussed
32	Introduction of filter bank diagram , its analysis phase and synthesis phase , perfect reconstruction in the filter bank	Seminar , lecture	The construction of filter bank diagram will be made clear
33	Lemma 3.15	Lecture	
34	The iteration steps in the construction of filter bank diagram	lecture	Particular cases will be discussed to make the concept clear
35	Introduction of $p^{\text{th}}$ stage wavelet filter sequence	lecture	Particular cases $p=2$ , $p=3$ will be discussed
36	The derivation of the output of the $p^{\text{th}}$ stage filter bank using down sampling operators	lecture	Particular cases will be discussed
37	The theory used for the reconstruction of the filter bank, the diagram representing the reconstruction phase using up sampling operators	Lecture , group discussion	Particular cases like $p=2$ , $p=3$ etc . will be discussed
38	Lemma 3.18	lecture	
39	Definition of $D^l$ and $U^l$ Corollary 3.19	Lecture , assignment	The definitions will be made clear through examples

40	Introduction of $f_i, g_i$ Definition 3.20	Lecture	Examples will be discussed to make the definitions clear
41	Lemma 3.21	lecture	
42	Lemma 3.22	lecture	
43	Introduction of $p^{\text{th}}$ stage wavelet basis for $l^2(\mathbb{Z}_N)$	lecture	
44	Lemma 3.24	Lecture, seminar	
45	Orthogonal direct sum of two subspaces of an inner product space and a problem from exercise	Assignment , lecture	Separate examples will be discussed
46	Lemma 3.26	lecture	
47	The theorem saying that a $p^{\text{th}}$ stage wavelet filter sequence can produce a $p^{\text{th}}$ stage wavelet basis . Theorem 3.27	lecture	
48	A new symbolic representation of the $p^{\text{th}}$ stage wavelet basis	lecture	
49	The folding lemma	Seminar , lecture	
50	Corollary 3.31	lecture	
51	The $p^{\text{th}}$ stage Haar system	lecture	
52	The $p^{\text{th}}$ stage Shannon wavelet basis for $l^2(\mathbb{Z}_N)$	Lecture ,seminar	
53	The $p^{\text{th}}$ stage real Shannon wavelet basis	Lecture	
54	Daubechies's D6 wavelets on $\mathbb{Z}_N$	Seminar , lecture	
55	The Hilbert space $l^2(\mathbb{Z})$ , Cauchy – Schwarz inequality ,and triangle inequality	Assignment , lecture	A comparative study will be done between $l^2(\mathbb{Z}_N)$ and $l^2(\mathbb{Z})$
56	Definition of Cauchy sequence , Complete inner product space, Hilbert space , symmetric partial sum , the convergence of the series in the form $\sum_{n \in \mathbb{Z}} w(n)$	Seminar , lecture	Some examples will be discussed
57	Lemma 4.7	lecture	
58	Lemma 4.8	lecture	

59	Complete orthonormal system , examples	Lecture , assignment	
60	Theorem 4.10	lecture	
61	Lemma 4.11	lecture	
62	Definition of $S_A$ and $P_S(f)$	Seminar , lecture	
63	Lemma 4.14	Lecture , assignment	
64	The Hilbert space $L^2([-\pi, \pi])$ , Cauchy – Schwarz inequality and triangle inequality	Lecture , seminar	
65	Introduction of $L^1([-\pi, \pi])$ , $L^2([-\pi, \pi])$ is a proper subset of $L^1([-\pi, \pi])$	Lecture	Example for the inclusion and example for the proper inclusion will be discussed
66	Introduction of the trigonometric system, and proving that it is an orthonormal set	Lecture	
67	Lemma 4.18	Lecture	
68	Lemma 4.19	Lecture	
69	Theorem 4.20	Assignment	
70	Theorem 4.21	Lecture	
71	The trigonometric system is complete	Lecture	
72	Definition 4.23 and corollary 4.24	Seminar , lecture	
73	Bounded linear transformation between Hilbert spaces and lemma 4.26	Lecture	
74	Introduction of translation operator and translation invariant linear transformation	Lecture	
75	Theorem 4.28	Lecture , seminar	
76	FOURIER TRANSFORM , INVERSE FOURIER TRANSFORM and convolution on $l^2(\mathbb{Z})$	Lecture	Additional problems will be solved
77	Lemma 4.31	Lecture	
78	Definition of summable sequences and the normed space $l^1(\mathbb{Z})$	Seminar, lecture	

79	Lemma 4.34	Lecture	
80	Lemma 4.35	Seminar	
81	The translation operator $R_k$ and translation invariant linear transformation on $l^2(Z)$ , example 4.37	Lecture	
82	The delta function and lemma 4.39	Lecture , assignment	
83	Definition of conjugate reflection of $z$ and lemma 4.41	Lecture , assignment	
84	Lemma 4.42	Lecture	
84	$U(z)$ , $D(z)$ , $U^m(z)$ , $D^m(z)$ and their properties	Lecture , assignment	Particular cases will be discussed , extra problems will be discussed
86	First stage wavelet system for $l^2(Z)$ , system matrix of $u$ and $v$ belonging to $l^2(Z)$	Lecture , seminar	
87	Theorem 4.46	Lecture	
88	Lemma 4.47	Lecture	
89	Lemma 4.48 , corollary 4.49	Lecture	
90	Lemma 4.50	Lecture	
91	Lemma 4.51	Seminar , lecture	
92	Introduction of $p^{\text{th}}$ stage wavelet system for $l^2(Z)$ and theorem 5.53	Lecture	
93	Introduction of a complete orthonormal system for $l^2(Z)$ , Theorem 4.55	Lecture , group discussion	
94	Haar wavelets on $Z$	Seminar , lecture	
95	Daubechies's $D_6$ wavelets on $Z$	Group discussion	
96	Problems from exercises	Group work	

## COURSE PLAN

### FUNCTIONAL ANALYSIS

#### COURSE OBJECTIVES

- \*To study basics of Functional analysis
- \*To identify the application of functional analysis in other disciplines
- \*To equip the students for NET
- \*To identify counter examples
- \*To improve the problem solving ability

Sessions	Topics to be taught	Method of teaching and evaluation	Remark
1	Fundamentals of metric space	Lecture , discussion ,seminar , assignment	Bridge course
2	Vector space and its properties	Lecture , problem solving , assignment	Special importance will be given to counter examples and NET oriented problems
3	Normed space , Banach space and their properties	Lecture , seminar , interaction	
4	Problems based on Normed spaces and Banach spaces	Lecture , Group wise problem solving	Importance will be given to counter examples
5	Further properties of Normed spaces,Problems	Lecture, interaction, assignment	Counter examples and NET oriented questions will be discussed
6	Finite dimensional Normed spaces and sub spaces	Lecture, seminar, assignment	
7	Problems based on finite dimensional Normed spaces and subspaces	Lecture,Individual problem solving,	Counter examples will be discussed

8	Compactness and finite dimension	Lecture, discussion ,seminar , assignment	
9	Problems based on compactness	Lecture ,groupwise problem solving , assignments	NET oriented problems will be discussed
10	Introduction of linear operators , their properties	Lecture , seminar , assignment,discussion	
11	Problems based on linear operators	Lecture, assignment, discussion	Counter examples will be discussed
12	<b>TEST PAPER ON MODULE -1</b>		
13	Bounded and continuous linear operators and their properties	Lecture , group discussion,assignment ,	
14	<b>Problems based on bounded linear operators</b>	Lecture , seminar, group problems	Counter examples and NET oriented problems will be discussed
15	<b>Linear functionals</b>	Lecture, seminar , assignment , discussion	

16	<b>FIRST INTERNAL</b>		
17	Bounded linear functional and its properties	Lecture, interaction, group discussion	
18	<b>Problems based on</b> bounded linear functionals	Lecture, groupwise problem solving, assignment	Counter examples will be discussed
19	<b>Linear operators and functionals</b> on a finite dimensional space	Lecture, seminar, assignments.	
20	Problems based on linear operators and functionals on finite dimensional spaces	Lecture, assignment, group wise problem solving	Counter examples will be discussed
21	<b>Normed spaces</b> of operators and functionals,	Lecture, seminar assignment,	
22	Examples of dual spaces	Lecture, assignment	
23			
24	<b>TEST PAPER ON MODULE -2</b>  Inner product spaces, Hilbert spaces	Lecture, seminar, assignment	



25	Examples and problems on inner product spaces and Hilbert spaces	Lecture , assignment discussion	Counter examples and NET based questions will be discussed
26	Further properties of inner product spaces,problems	Lecture , assignments ,group discussion	Counter examples will be discussed
27	Orthogonal complement and direct sum, problems	Lecture , seminar ,group problem solving	Counter examples will be discussed
28	<b>Orthogonal sets and sequences</b>	Lecture , seminar, interaction	
29	Bessel inequality, Gram-schmidt process for orthonormalisation , problems	Lecture , assignment, individual problem solving	
30	Total orthonormal sets and sequences, problems	Lecture , seminar , assignments	
31	Riesz's theorem	Lecture, assignment	
32	Sesqui linear functional and Riesz representation theorem	Lecture , discussion, assignment	

33	Problems based on Riesz theorem and Riesz representation theorem	Lecture, assignments ,	Counter examples will be discussed
34	Hilbert adjoint operator and its properties	Lecture , assignments,interaction	
35	Problems based on Hilbert adjoint operator	Lecture , assignment ,group problems	NEToriented questions will be discussed
36	Self adjoint , Normal ,Unitary operators	Lecture , seminar , assignment	
37	Problems based on self adjoint , normal , unitary operators	Lecture , individual problem solving	NETbased questions will be discussed
38	<b>TEST PAPER ON MODULE – 3</b>		
39	Zorn's Lemma and its applications	Lecture, assignment	
40			

41	Hahn Banach theorem for real vector space	Lecture, seminar, group discussion	
42	Generalised Hahn Banach theorem , problems	Lecture, assignment	
43	Hahn Banach theorem for Normed space	Lecture, interaction	NETbased problems will be discussed
45	Problems on Hahn banach theorems	Lecture, assignment	
46	Adjoint operator , relation between adjoint operator and Hilbert adjoint	Lecture Lecture , seminar, assignments	NET based problems will be discussed
47	Reflexive spaces , canonical mapping, important theorems, problems	Lecture	
48	Bairs category theorem , Uniform boundedness theorem	Lecture , group problems , interaction	N ET based questions will be discussed
49	Problems based on Uniform boundedness theorem		
50	<b>Conclusion of the course</b>		

51	<b>SECOND INTERNAL</b>  <b>Model examination</b>		<b><i>DETAIL STUDY MATERIAL WILL BE PROVIDED FOR EACH MODULE</i></b>
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