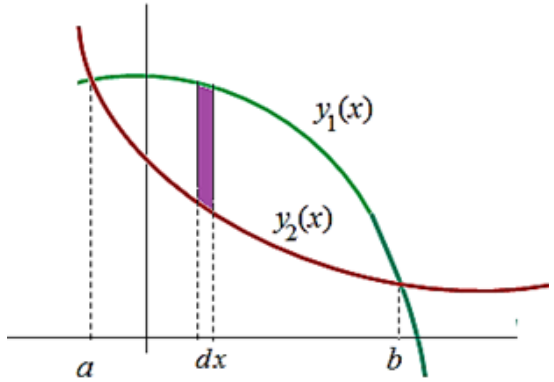
CALCULUS

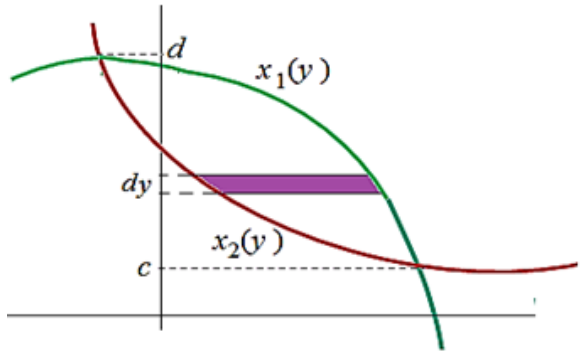
Applications Of The Definite Integral

- **The area between two curves**
- **The Volume of the Solid of revolution**

AREA BETWEEN CURVES

$$dA = \left\{\begin{pmatrix} outer \\ function \end{pmatrix} - \begin{pmatrix} inner \\ function \end{pmatrix}\right\} \text{dx}$$





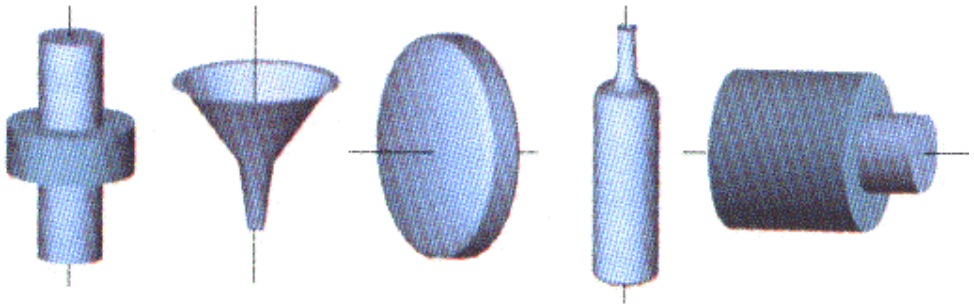
$$A = \int_a^b dA = \int_a^b [y_1(x) - y_2(x)]dx$$

$$A = \int_c^d dA = \int_c^d [x_1(y) - x_2(y)]dy$$

VOLUME OF A SOLID OF REVOLUTION

- DISC METHOD
- WASHER METHOD
- SHELL METHOD

If a region in the plane is revolved about a given line, the resulting solid is a solid of revolution, and the line is called the axis of revolution.

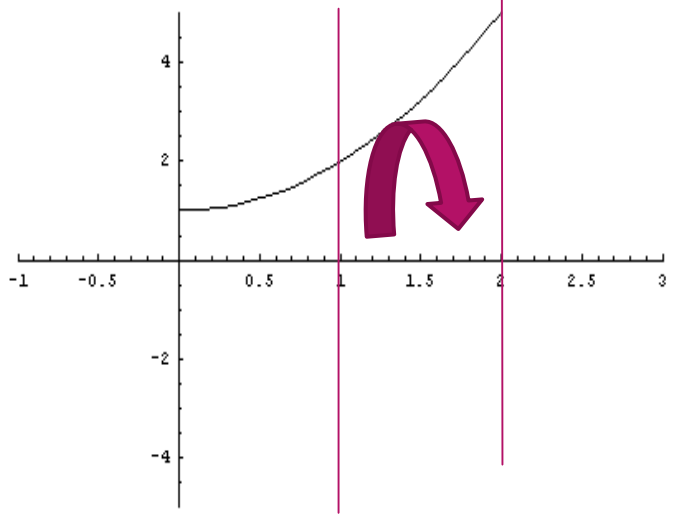


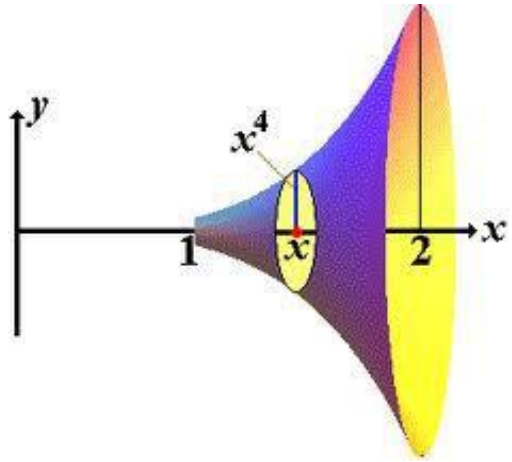
Solids of Revolution

DISC METHOD

- The volume of the solid generated by a region under *f(x)* bounded by the x-axis and vertical lines *x=a* and *x=b*, which is revolved **about the x-axis** is

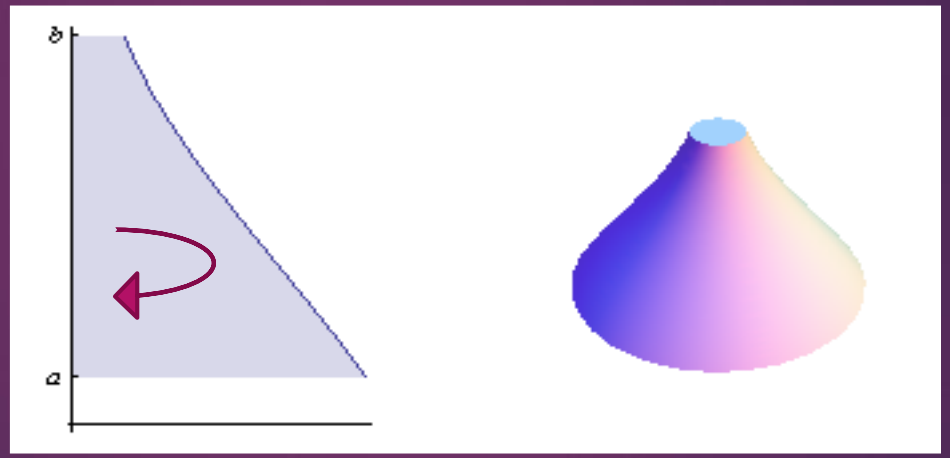
$$V = \pi \int_a^b \big(f(x)\big)^2 \, dx$$





The volume of the solid generated by a region under *f(y)* (to the left of *f(y)* bounded by the y-axis, and horizontal lines *y=c* and *y=d* which is revolved **about the y-axis**

$V = \pi \int_c^d (f(y))^2 dy$

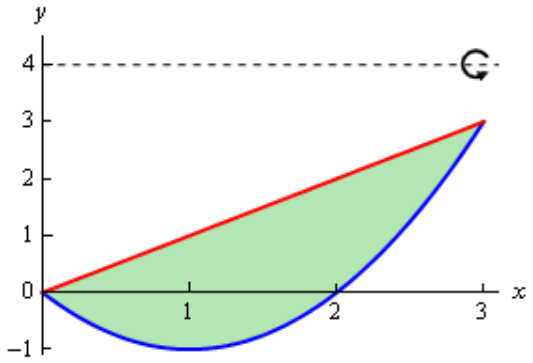


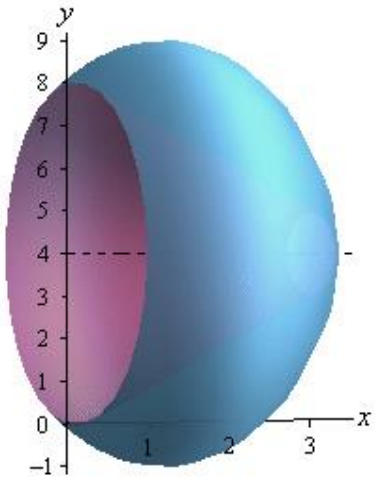
WASHER METHOD

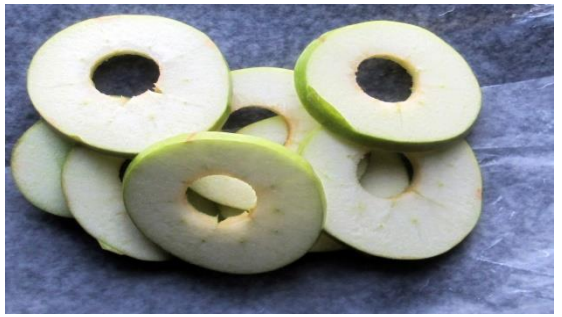
This is an extension of the disc method.

- The volume of the solid generated by a region between *f(x)* and *g(x)* bounded by the vertical lines x=a and x=b, which is revolved about the x-axis is

$$V = \pi \int_a^b \left|((f(x))^2 - (g(x))^2\right| dx$$

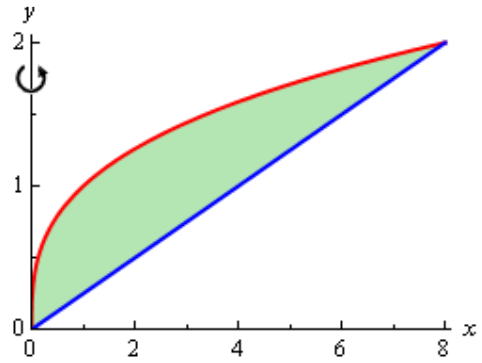


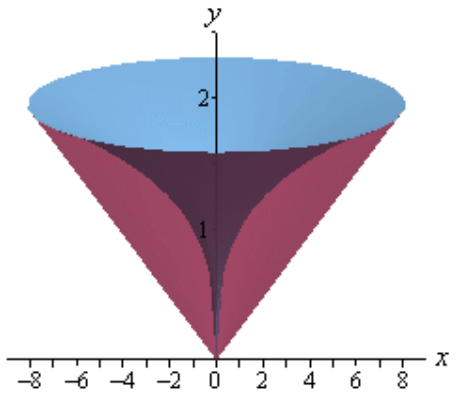




- The volume of the solid generated by a region between *f(y)* and *g(y)* bounded by the horizontal lines y=c and y=d which is revolved about the y-axis is

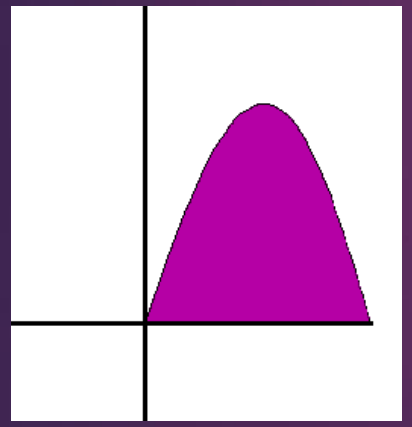
$$V = \pi \int_c^d \left| \left(f(y)\right)^2 - \left(g(y)\right)^2 \right| dy$$

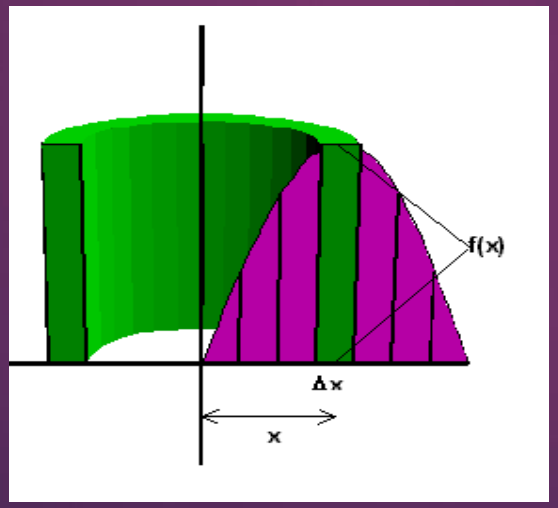


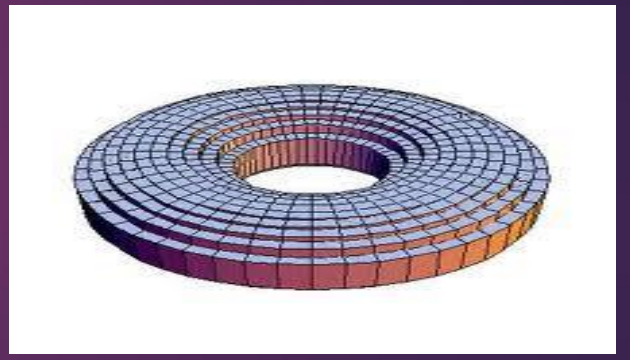


SHELL METHOD

The shell method is a method of calculating the volume of a solid of revolution when integrating along an axis parallel to the axis of revolution.







- The volume of the solid generated by a region bounded by the vertical lines x=*a* and x=*b*, which is revolved about the y-axis is

  $V = 2\pi \int_a^b xf(x)dx$

- The volume of the solid generated by a region bounded by the y-axis, and horizontal lines *y*=c and *y*=*d* which is revolved about the x-axis is

  $V = 2\pi \int_c^d yf(y)dy$

THANK YOU