HOMOGENEOUS LINEAR EQUATION WITH CONSTANT COEFFICIENTS

Higher Order Linear Differential Equations

CASE 1: DISTINCT REAL ROOTS:

Consider the nth-order homogenous linear differential equation with constant coefficients. If the auxiliary equation has distinct real roots $m_1, m_2, \ldots, m_n$ the general solution is

$$y = c_1 e^{m1x} + c_2 e^{m2x} + \ldots\ldots\ldots + c_n e^{mnx}$$

Where $c_1, c_2, \ldots$.. are constants

Solve

$$\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + \frac{dy}{dx} + 6y = 0$$

Answer:

The auxiliary eqaution is $m^3-4m^2+m+6=0$

The roots of this equation is $m=-1,2,3$

The general solution is

$$y = c_1 e^{-1x} + c_2 e^{2x} + c_3 e^{3x}$$

CASE 2: REPEATED REAL ROOTS:

Consider the nth-order homogenous linear differential equation with constant coefficients. If the auxiliary equation has real roots m occurring k times the general solution is

$$y = (c_1 + c_2 x + c_3 x^2 + \dots\dots\dots + c_k x^{k-1})e^{mx}$$

Where $c_1, c_2, \dots..$ are constants

Solve

$$\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 18y = 0$$

Answer:

The auxiliary eqaution is $m^3-4m^2-3m+18=0$
The roots of this equation is $m=3,3,-2$

The general solution is

$$y=(c_1+c_2x)e^{3x}+c_3e^{-2x}$$

CASE 3: CONJUGATE COMPLEX ROOTS:

Consider the nth-order homogenous linear differential equation with constant coefficients. If the auxiliary equation has conjugate complex roots

(a+bi) and (a-bi)

the general solution is

$$y = e^{ax}(c_1 \sin bx + c_2 \cos bx)$$

Where $c_1, c_2, \dots$.. are constants

Solve

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = 0$$

Answer:

The auxiliary eqaution is $m^2-6m+25=0$

The roots of this equation is

$$m=3+4i \text{ and } 3-4i$$

The general solution is

$$y = e^{3x}(c_1 \sin 4x + c_2 \cos 4x)$$