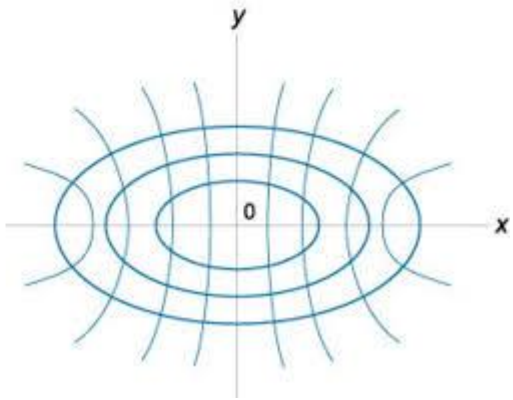
APPLICATIONS of FIRST ORDER DIFFERENTIAL EQUATIONS

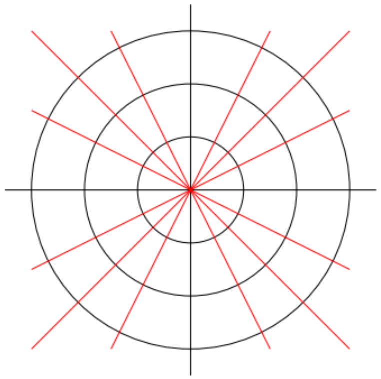
ORTHOGONAL TRAJECTORIES

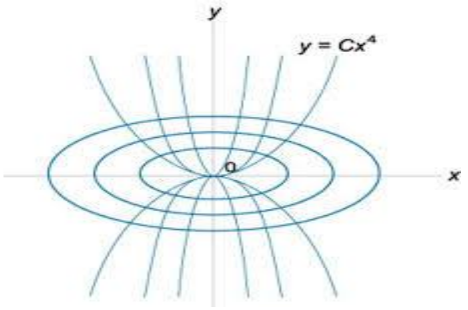
Definition:

Let F(x;y;c) = 0 (1)

be a given one-parameter family of curves in the xy-plane. A curve that inter-sects the curves of the family(1)at right angles is called an orthogonal trajectory of the given family


y
0
x




y
y = Cx⁴
O
x

WORKING METHOD FOR FINDING THE ORTHOGONAL TRAJECTORY

- Method:
- Step 1:- To fInd the orthogonal trajectories of a family of curves(1);first differentiate equation(1)implicitly with respect to x and obtain the differential equation of the given family of curves(1).
- Step 2:-Eliminate the parameter c between the derived equation and the given equation(1).
- Step 3:-Let us assume that the resulting differential equation of the family(1)can be expressed in the form dy/dx=f(x;y).
- Step 4:-Since an orthogonal trajectory of the given family intersects each curve of given family at right angels, the slope of the orthogonal trajectory at(x; y) is -1/f(x;y). So, the differential equation of the family of orthogonal trajectories is

  dy/dx = -1/f(x;y).

PROBLEMS

Example 1)

Find the orthogonal trajectories of the family of parabola $y= cx^2$;where c is an arbitrary constant.

Solution. Differentiating $y=cx^2$ (1)

we obtain the differential equation $dy/dx = 2cx$ (2)

Substituting $c = y/x^2$ into (2)

we obtain $dy/dx = 2y/x$

which is the differential equation of the given family of parabolas. So $dy/dx= -x/2y$ (3) is the differential equation of the orthogonal trajectories of the given family(1). Solving (3) by separating variables we obtain

$2y^2+x^2=c^2$ where c is a constant.

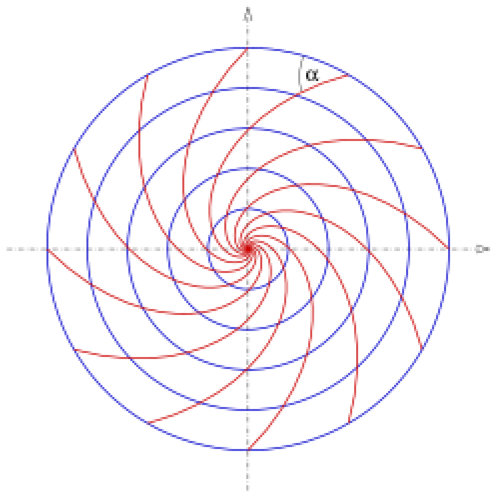
OBLIQUE TRAJECTORIES

Definition:- Let F=(x;y;c) = 0 be a one parameter family of curves. A curve which intersects the curves of the given family at a constant angle **a ≠ 90°** is called an **oblique trajectory** of the given family.

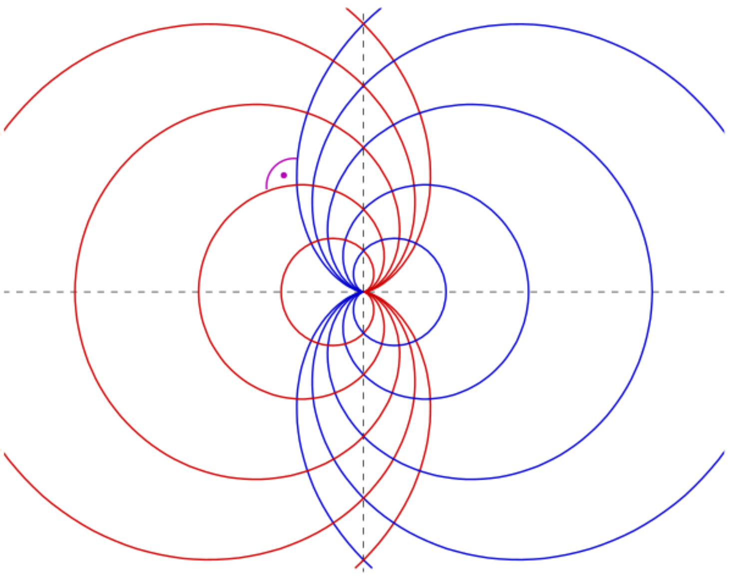
Suppose the differential equation of the given family is dy/dx = f(x;y) .

Then the differential equation of a family of oblique trajectories is given by

$$dy/dx = \frac{f(x,y) + \tan a}{1 - f(x,y)\tan a}$$



α



Find the family of oblique trajectories that intersect the family of circles $x^2+y^2=c^2$ at angle 45°.

Solution.

From $x^2+y^2=c^2$ (1)

we get $2x + 2y\, dy/dx = 0$

$dy/dx = -x/y$

The differential equation of the family of oblique trajectories are given by $\frac{dy}{dx} = \frac{\frac{-x}{y}+1}{1+\frac{x}{y}} = \frac{y-x}{y+x}$ (2)

It is clear that eqn (2) is a homogeneous differential equation. Applying the transformation $y= vx$, we obtain separable differential equation $x\frac{dv}{dx} + v = \frac{x(v-1)}{x(v+1)}$

$$\frac{v+1}{v^2+1}dv = -\frac{dx}{x}$$

Hence the solution is $\ln c^2(x^2+y^2)+2 \arctan \frac{y}{x}=0$.

THANK YOU