GRAPH THEORY MORE DEFINITIONS

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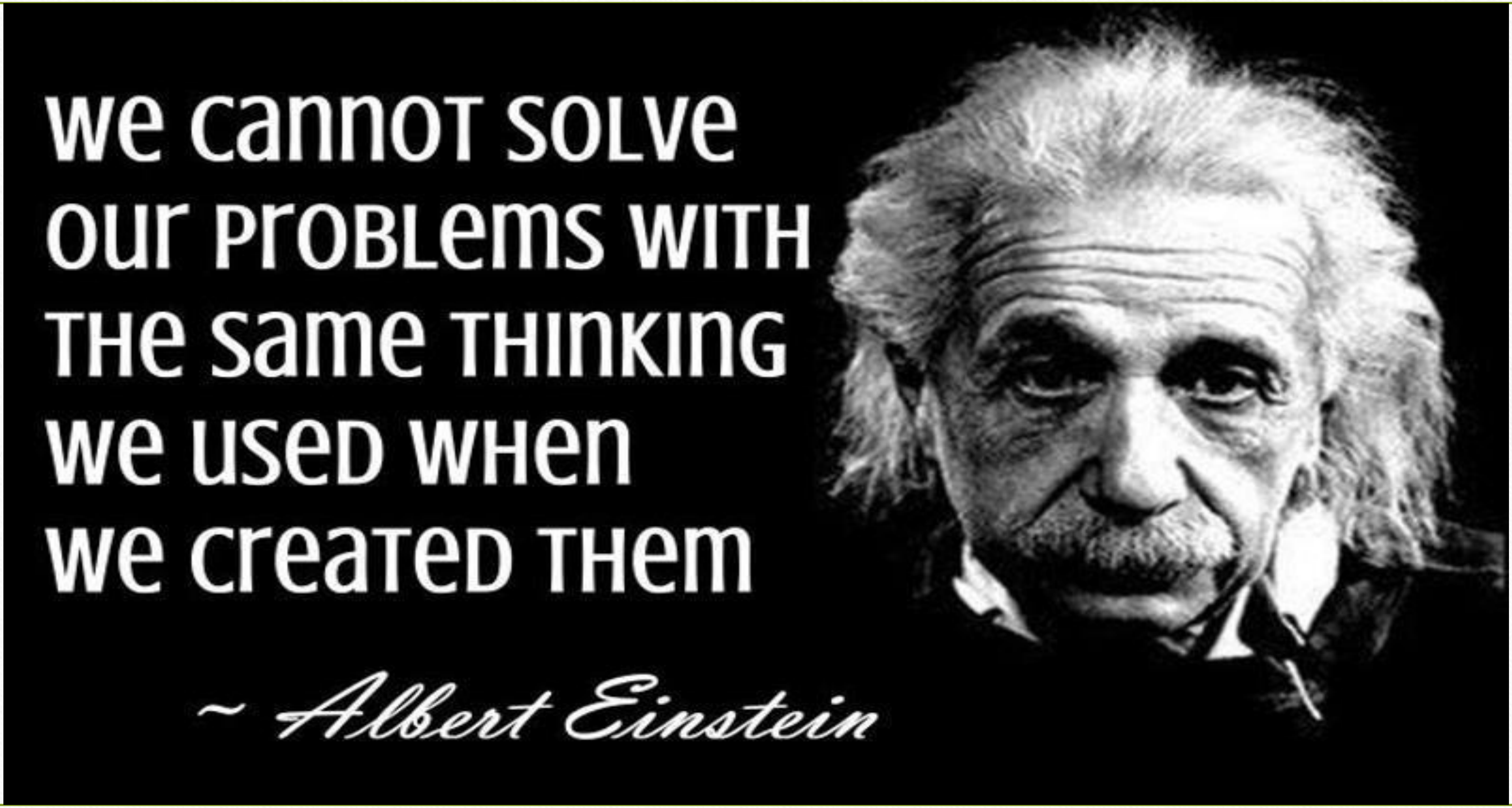

We cannot solve
our problems with
the same thinking
we used when
we created them
~ Albert Einstein

- Let *H* be a graph with vertex set *V(H)* and edge set *E(H)* and, similarly, let *G* be a graph with vertex set *V(G)* and edge set *E(G).* Then we say that H is a subgraph of *G* if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$. In such a case, we also say that *G* is a supergraph of *H.*

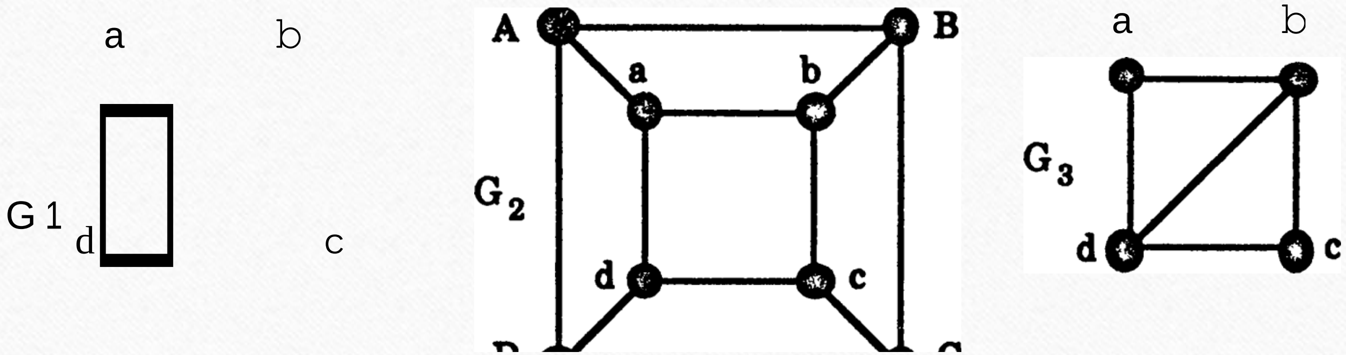


Figure 1.22: $G_1 \subseteq G_2$, $G_1 \subseteq G_3$ but $G_3 \not\subseteq G_2$.

DEFINITION

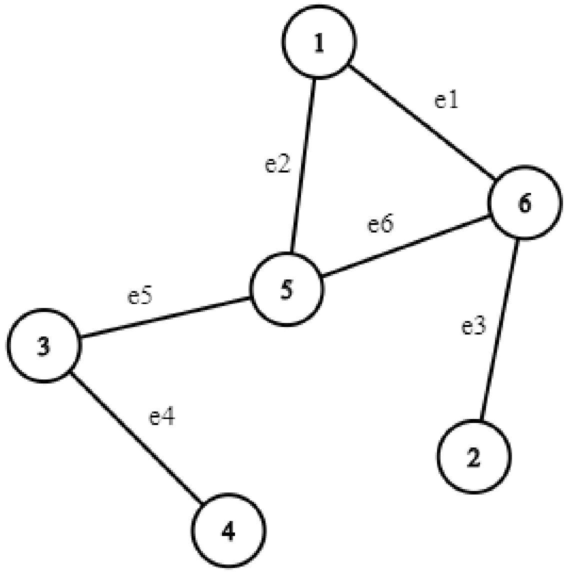
- Any graph *isomorphic* to a subgraph of G is also referred to as a **subgraph** of G.

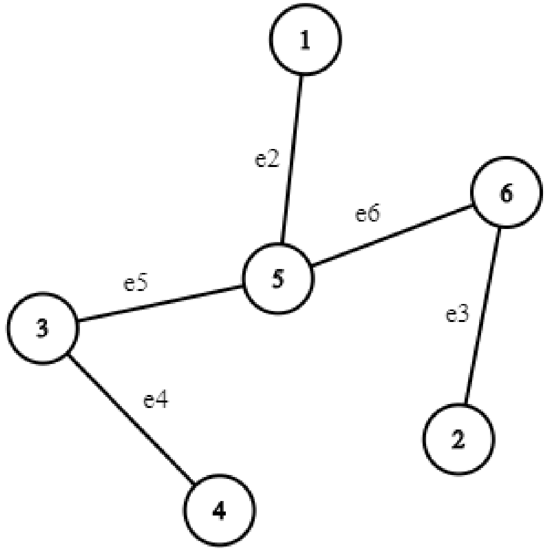
If H is a subgraph of G we write $H \subseteq G$. If $H \subseteq G$ and $H \neq G$ ie $V(G) \neq V(H)$ or $E(G) \neq E(H)$ then we say that H is a proper subgraph of G.

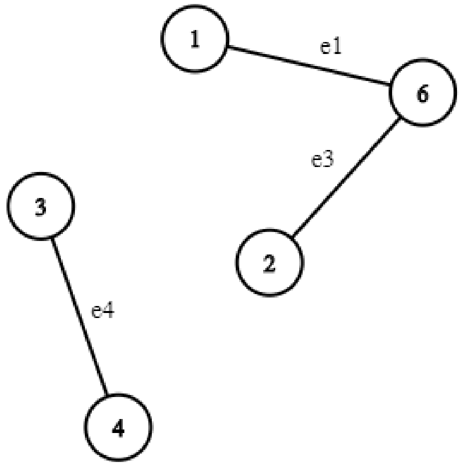
- A spanning subgraph (or spanning supergraph) of G is a subgraph (or supergraph) H with $V(H) = V(G)$, i.e., H and G have exactly the same vertex set.
- Example : G1 is a proper spanning subgraph of G3.

Vertex deleted subgraph.

- If $G = (V, E)$ and V has at least two elements (i.e., G haS at least two vertices), then for any vertex v of G, $G - v$ denotes the subgraph of G with vertex set $V - \{v\}$ and whose edges are all those of U which are not incident with v, i.e., $G - v$ is obtained from G by removing v and all the edges of G which have v as an end. $G - v$ is referred to as a **vertex deleted subgraph**.
- If $G = (V, E)$ and e is an edge of G then G - e denotes the subgraph of G having V as its vertex set and $E - \{e\}$ as its edge set, i.e., G - e is obtained from G by removing the edge e, (but not its endpoint(s)). G - e is referred to as an edge deleted subgraph.







Example

G

G-{e1}

edge e1 deleted graph

G-{5}

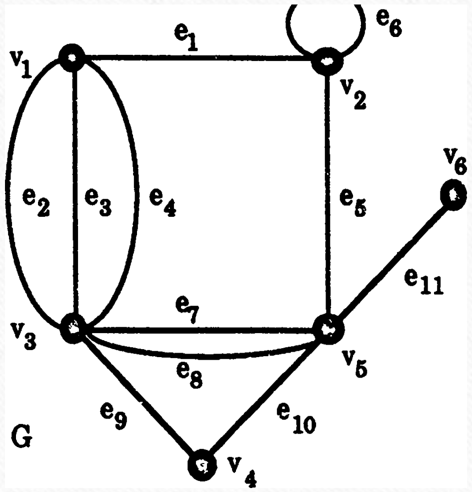
vertex 5 deleted graph

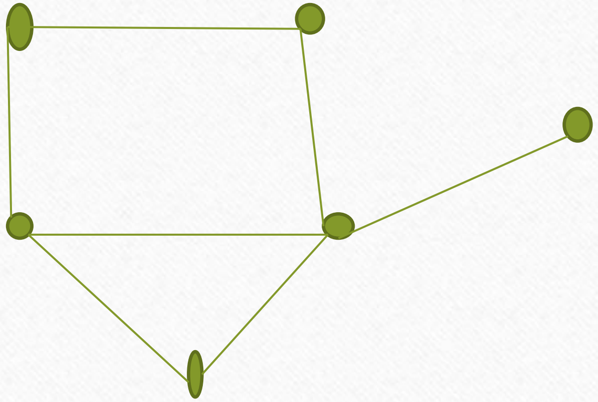
VERTEX DELETED AND EDGE DELETED SUBGRAPHS

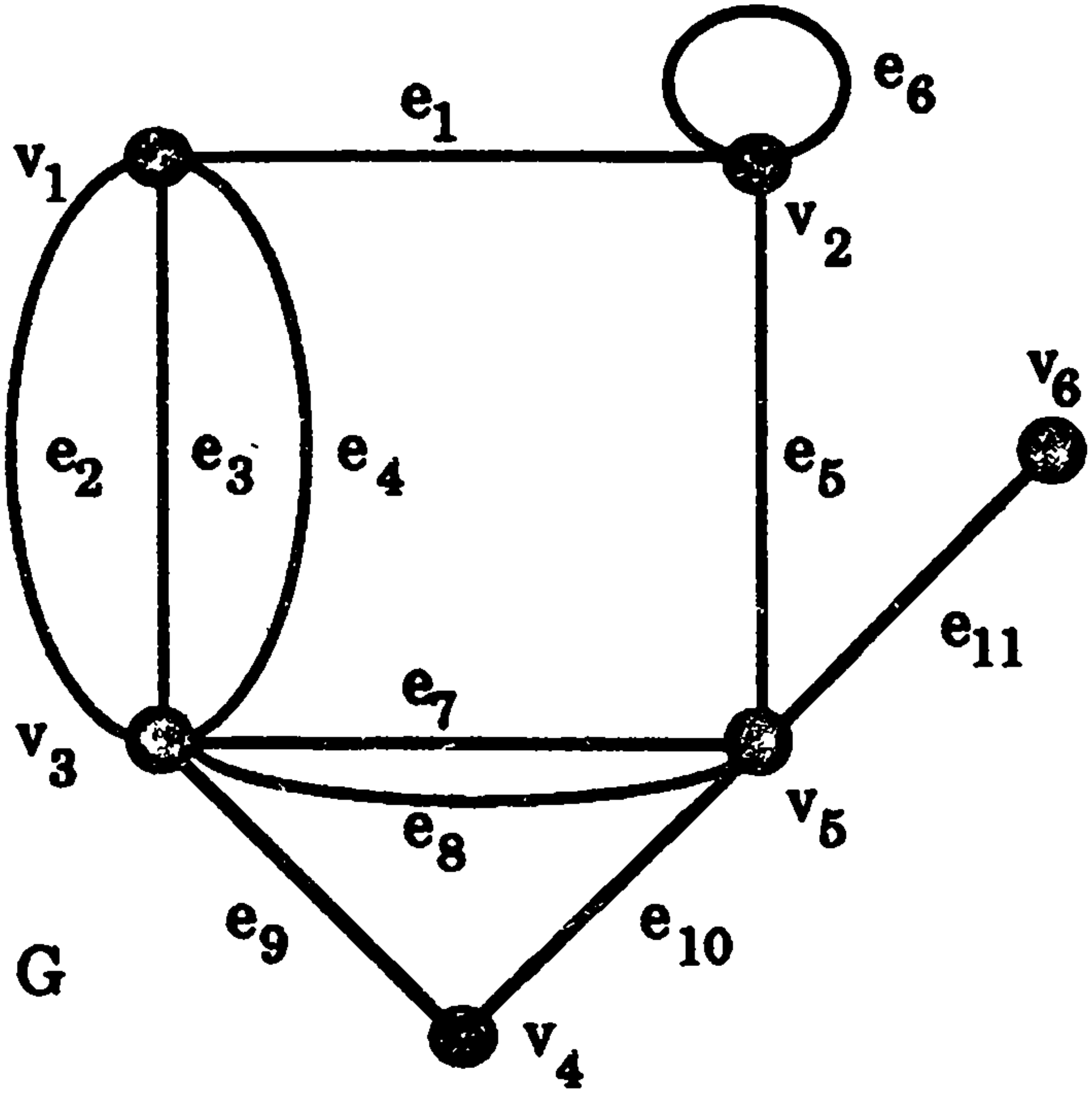
- If $G = (V, E)$ and U is a proper subset of V then $G - U$ denotes the subgraph of G with vertex set $V - U$ and whose edges are all those of G which are not incident with any vertex in U.
- If F is a subset of the edge set E then $G - F$ denotes the subgraph of G with vertex set V and edge set $E - F$, i.e., obtained by deleting all the edges in F, but not their endpoints.
- G-U and G-F are also referred to as vertex deleted and edge deleted **subgraphs** (respectively).

Underlying simple graph

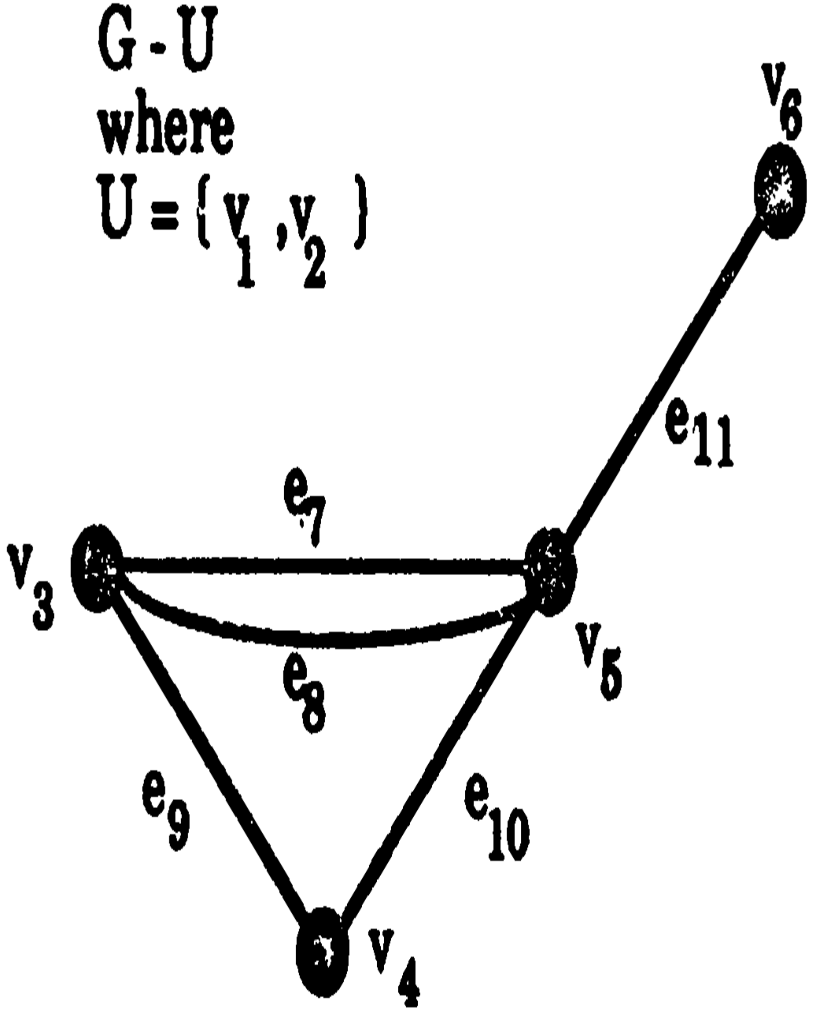
By deleting from a graph *G* all loops and in each collection of parallel edges all edges but one in the collection we obtain a simple spanning subgraph of *G*, called the **underlying simple graph** of *G*.



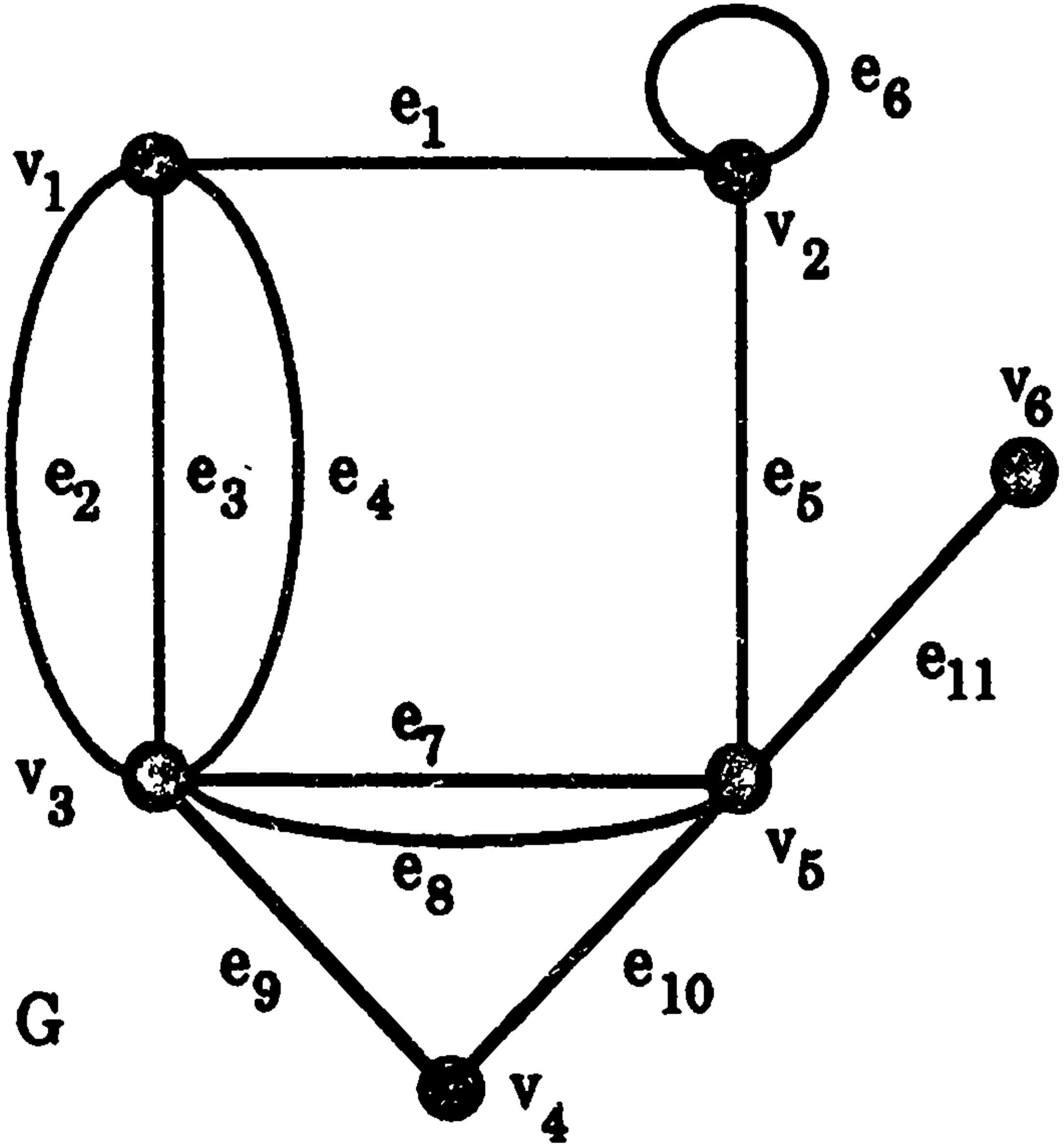




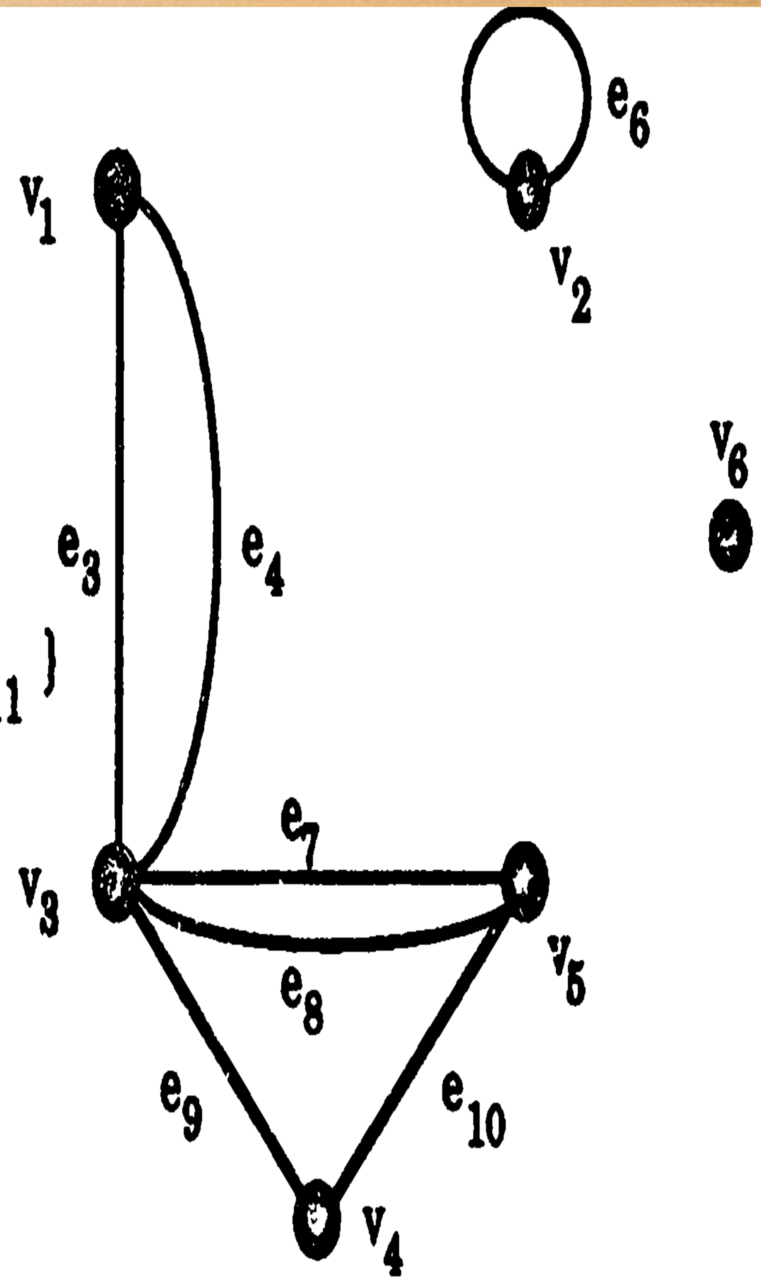
v1
e1
e6
v2
e2
e3
e4
e5
v6
e11
e7
v3
v5
e8
e9
e10
G
v4



G - U
where
U = { v1 , v2 }
v6
e11
e7
v3
v5
e8
e9
e10
v4



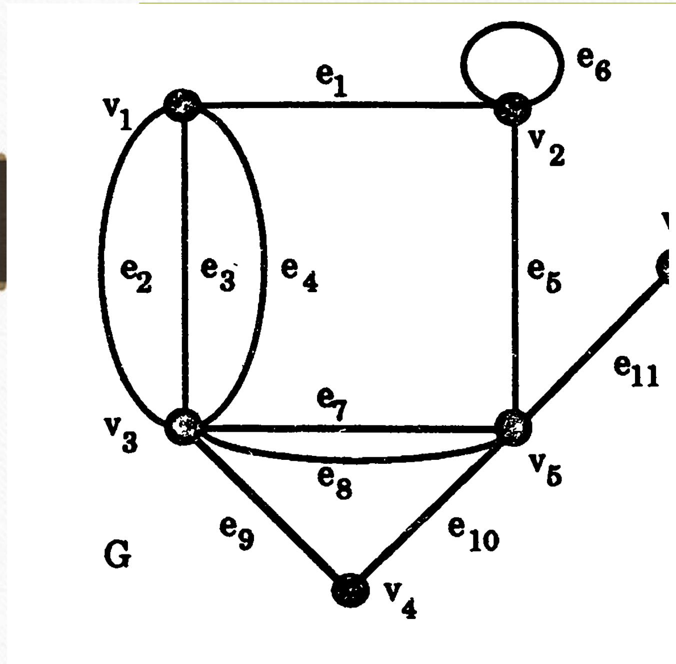
G - F
where
$F = \{e_1, e_2, e_5, e_{11}\}$

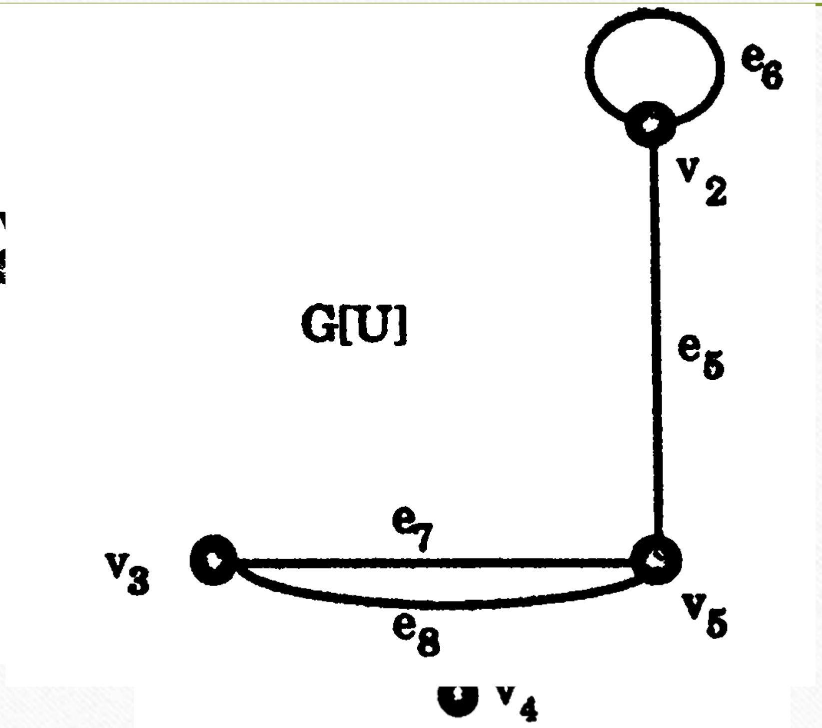


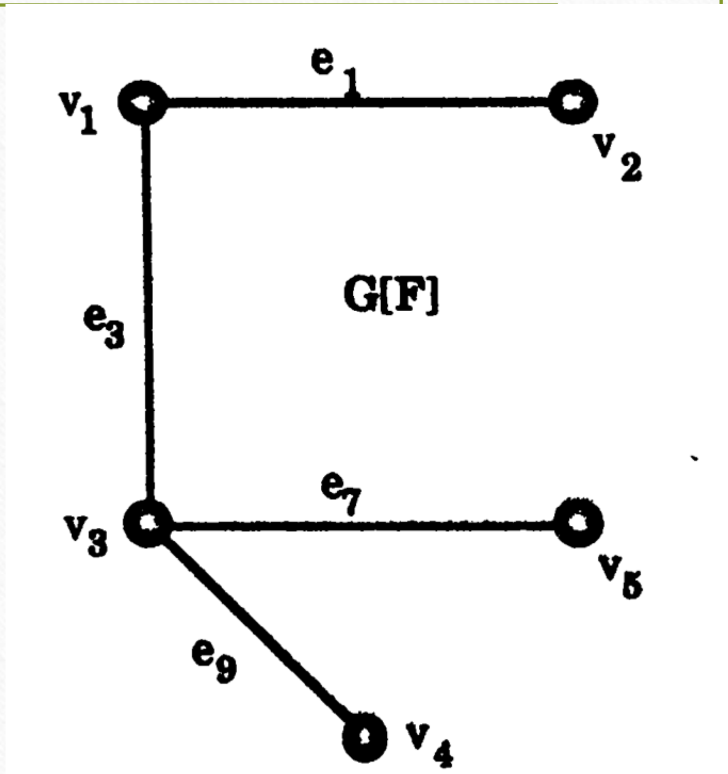
INDUCED SUBGRAPHS

- *If* U is a nonempty subset of the vertex set V of the graph G then the subgraph G[U] of G induced by U is defined to be the graph having vertex set U and edge set consisting of those edges of G that have both ends in U.
- Similarly if F is a nonempty ·subset of the edge set E of G then the subgraph G[F] of G induced by F is the graph whose vertex set is the set of ends of edges in F and whose edge set is F.

For the graph G of Figure 1.24, taking U = {v2, v3,v5} and F = {e1, e3, e7,e9}we get G[U] and G[F]







- Two subgraphs G1 and G_2 of a graph *G* are said to be disjoint if they have no vertex in common, and edge disjoint if they have no edge in common.

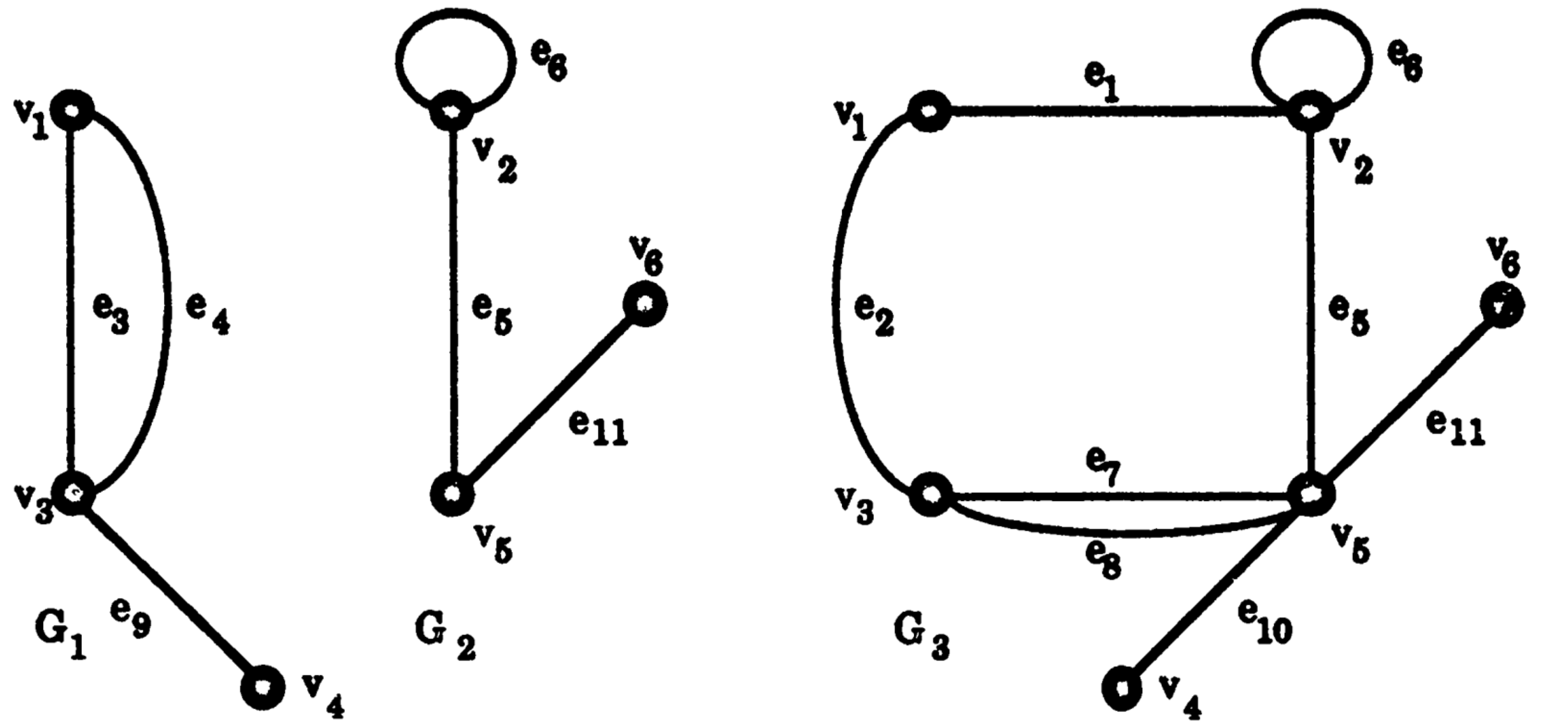


Figure 1.26: G_1 and G_2 are disjoint and G_1 and G_3 are edge disjoint.

Union of Two Subgraphs

- Given two subgraphs G1 and G2 of G, the union G_1 U G_2 is the subgraph of *G* with vertex set consisting of all those vertices which are in either G_1 or G2 (or both) and with edge set consisting of all those edges which are in either G1 or G2 (or both); symbolically
- V{G1 U G2) = V(G1) U V(G2),
- *E(G1* U G2) = E(G1) U E(G2).

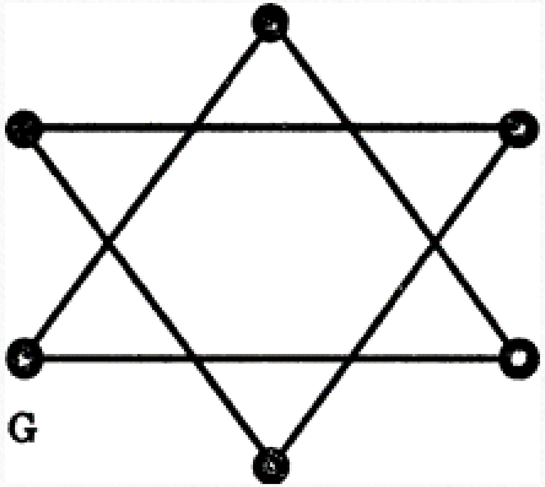
Intersection of two subgraphs

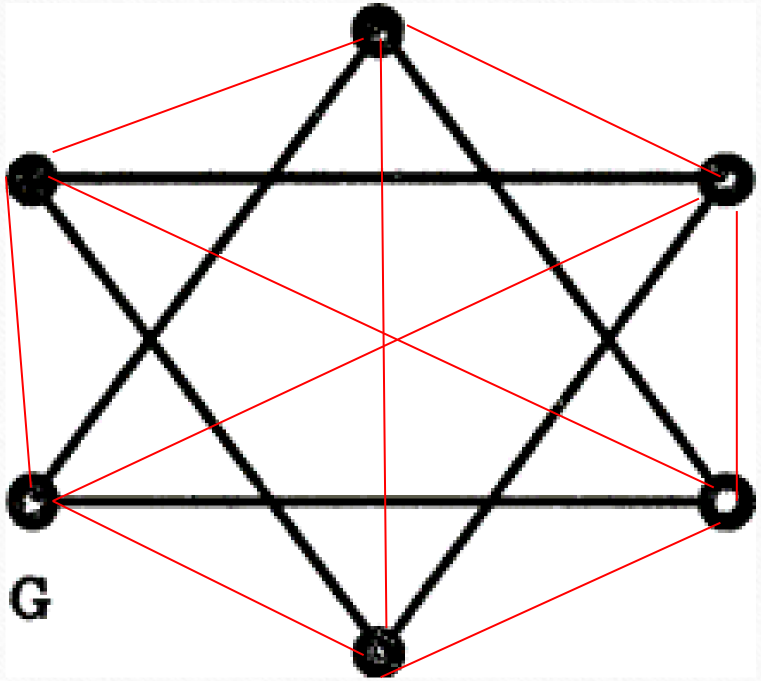
- If G1 and G2 are two subgraphs of *G* with at least one vertex in common then the **intersection** G1n G2 is given by
- V(G1 n G2) = V(G1) n V(G2),
- E(G1 n G2) = E(G1) n E(G2)-

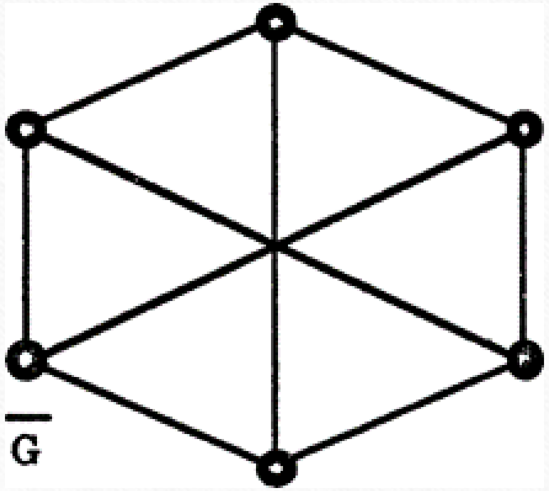
THE COMPLEMENT OF A GRAPH

- Let G be a simple graph with n vertices. The complement $\bar{G}$ of G is defined to be the simple graph with the same vertex set as G and where two vertices u and v are adjacent precisely when they are *not* adjacent in G. Roughly speaking then, the complement of G can be obtained from the complete graph Kn by "rubbing out" all the edges of $\bar{G}$

G K_6 $\bar{G}$







Self-complementary graph

- A simple graph is called **self-complementary** if it is isomorphic to its own complement.

ASSIGNMENT

18UMAT6440	COMMON TO ALL	1.5.1
18UMAT6401	HARIKRISHNAN T R	1.5.2
18UMAT6402	ARCHANA S	1.5.3
18UMAT6404	MITHALI S KUMAR	1.5.4
18UMAT6405	JOFFIN C CLEMENT	1.5.5

THANK YOU

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