Polynomial Equations and Factoring

SANIL JOSE
DEPARTMENT OF MATHEMATICS

Adding and Subtracting Polynomials

monomial: a real number, a variable, or the product of a real number and one or more variables with whole number exponents
3,3x,$3x^2$

polynomial: a monomial or the sum of monomials 3x, 3+3x+$3x^2$,5x^0

Adding and Subtracting Polynomials

degree of a polynomial: the monomial in the polynomial with the greatest exponent

standard form of a polynomial: the degree of the monomial terms are in order from greatest to least

Adding and Subtracting Polynomials

leading coefficient: the constant for the first term in a polynomial expression written in standard form

Adding and Subtracting Polynomials

Core Concepts **Classifying Polynomials**

Degree	Name	Terms	Name
0	constant	1	monomial
1	linear	2	binomial
2	quadratic	3	trinomial
3	cubic	4 or more	polynomial
4	quartic		

Adding and Subtracting Polynomials

Examples

Write the expression in standard form. Determine the degree and the leading coefficient.

a) $-8 + 9x^2 - 2x$

b) $3 + 7a^3$

Adding and Subtracting Polynomials

Solutions

a) $9x^2 – 2x – 8$, 2^{nd} degree, 9 is leading coefficient
b) $7a^3 + 3$, 3^{rd} degree, 7 is leading coefficient

Adding and Subtracting Polynomials

Write the polynomial in standard form. Then classify the polynomial by its degree and number of terms.

c) $-b^2 + 2b^4 + 6b$

d) $12c^2$

Adding and Subtracting Polynomials

Solutions

c) $2b^4 – b^2 + 6b$, quartic trinomial

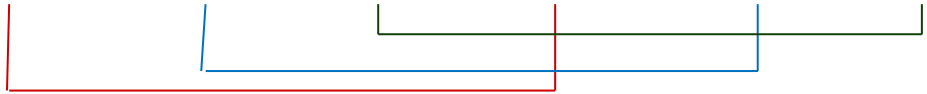
d) $12c^2$, quadratic monomial

Adding and Subtracting Polynomials

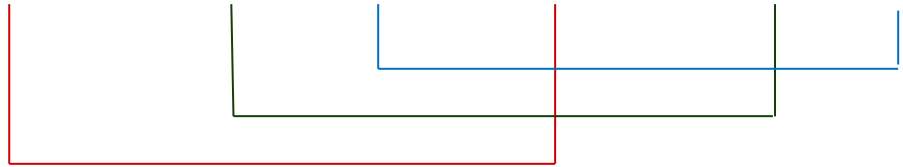
Find the sum.

e) $(3x^2 + 4x + 3) + (2x + 5x^2 + 1)$

$= (3x^2 + 4x + 3) + (5x^2 + 2x + 1) = 8x^2 + 6X + 4$



f) $(-5a^2 + a + 2) + (2a^2 - a - 9) = -3a^2 + 0a - 7$



Adding and Subtracting Polynomials

Find the difference.

g) $(3y^2 + 8) - (6y^2 + 4y - 2) = -3y^2 - 4y + 10$

h) $(12t^2 + 5t - 7) - (-4t^2 + 3t - 1) = 16t^2 + 2t - 6$

Multiplying Polynomials

I can multiply binomials and trinomials.

Multiplying Polynomials

FOIL (First Outer Inner Last) Method: in order to multiply binomials we can multiply first term with first term, outer with outer, inner with inner and last with last, then simplify

Multiplying Polynomials

Examples (space on page 211 in Student Journal)

Find the product.

a) (x + 8)(x + 2)=$x^2 + 8x + 2x + 16 = x^2 + 10x + 16$

b) (x – 5)(x + 1)=$x^2 + x - 5x - 5 = x^2 - 4x - 5$

c) (2x + 3)(x – 3)=$2x^2 - 6x + 3x - 9 = 2x^2 - 3x - 9$

Multiplying Polynomials

Find the product.

d) $(2x - 1)(5x - 4)$

e) $(x - 3)(x^2 - 4x - 4)$

Multiplying Polynomials

Solutions

d) $10x^2 – 13x + 4$

e) $x^3 – 7x^2 + 8x + 12$

Special Products of Polynomials

I can use the square of a binomial and sum and difference patterns.

Special Products of Polynomials

The square of a binomial is the square of the first term plus twice the product of the first and last term plus the square of the last term.

The product of the sum and difference of the same 2 terms is the difference of their squares.

Special Products of Polynomials

Examples

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$

Find the square.

1. $(2x+3)^2 = (2x)^2 + 2(2x)3 + 3^2 = 4x^2 + 12x + 9$
2. $(3x-5)^2 = (3x)^2 - 2(3x)5 + 5^2 = 9x^2 - 30x + 25$

Special Products of Polynomials

Find the product.

c) $(a + 8)(a – 8)$

d) $(4x + y)(4x – y)$

Special Products of Polynomials

Solutions

c) $a^2 – 64$

d) $16x^2 – y^2$

Solving Polynomial Equations in Factored Form

I can factor polynomials using the GCF and use the Zero-Product Property.

Solving Polynomial Equations in Factored Form

factored form: a polynomial written as a product of factors

Zero-Product Property: if the product of 2 factors equals 0, then at least 1 of the factors is equal to 0

Solving Polynomial Equations in Factored Form

roots: the solutions to a polynomial equation

repeated roots: when 2 or more roots for the equation are the same

greatest common factor (GCF): a monomial that divides evenly into each term

Solving Polynomial Equations in Factored Form

Examples

Solve the equation.

a) $3x(x - 6) = 0 \approx 3x = 0 \; or \; x - 6 = 0 \approx x = 0 \; or \; x = 6$

b)

$(x + 5)(x - 4) = 0 \approx x + 5 = 0 \; or \; x - 4 = 0 \approx x = -5 \; or \; x = 4$

Solving Polynomial Equations in Factored Form

Solve the equation.

c) $(4x + 5)(4x – 5) = 0$

d) $(c + 6)^2 = 0 \approx (c + 6)(c + 6) = 0 \approx c + 6 = 0 \; or \; c + 6 = 0 \approx c = -6 \; or \; c = -6$, c = -6 (repeated root)

e) $(a + 5)(a – 2)(a – 7) = 0$

Solving Polynomial Equations in Factored Form

Solutions

c) x = -5/4, 5/4

d) c = -6 (repeated root)

e) a = -5, 2, 7

Solving Polynomial Equations in Factored Form

Write the polynomial in factored form by factoring out the GCF.

f) $12x^3 + 3x^2 = 3x^2(4x + 1)$

g) $4x^4 + 24x^3 = 4x^3(x + 6)$

Solving Polynomial Equations in Factored Form

Solutions

f) $3x^2(4x + 1)$

g) $4x^3(x + 6)$

Solving Polynomial Equations in Factored Form

Solve the equation.

h) $4x^2 + 12x = 0$

i) $-10a^2 = 8a$

Solving Polynomial Equations in Factored Form

Solutions

h) $4x(x + 3) = 0$, $x = -3, 0$

i) $-10a^2 = 8a$, $-10a^2 - 8a = 0$, $-2a(5a + 4) = 0$, $a = -4/5, 0$

Factoring $x^2 + bx + c$

I can factor $x^2 + bx + c$.

Factoring $x^2 + bx + c$

Core Concepts

In order to factor a trinomial of the form $x^2 + bx + c$, we must find 2 numbers that multiply to be c and add to be b.

Factoring $x^2 + bx + c$

Examples

Factor.

a) $x^2 + 9x + 14 = 0$ $a + b = 9, ab = 14, a = 7, b = 2$

b) $x^2 - 12x + 27$, $a + b = -12, ab = 27, a = -9, b = -3$

c) $x^2 + 7x - 8$, $a + b = 7, ab = -8, a = 8, b = -1$

Factoring $x^2 + bx + c$

Solutions

a) $(x + 2)(x + 7)$
b) $(x - 9)(x - 3)$
c) $(x - 1)(x + 8)$

Factoring $ax^2 + bx + c$

I can factor $ax^2 + bx + c$.

Factoring $ax^2 + bx + c$

Examples

Factor the polynomial.

a) $4x^2 + 32x + 60 = 4(x^2 + 8x + 15), a = 5, b = 3$

b) $5x^2 + 15x + 10 = 5(x^2 + 3x + 2), a = 2, b = 1$

Factoring $ax^2 + bx + c$

Solutions

a) $4(x^2 + 8x + 15) = 4(x + 3)(x + 5)$

b) $5(x^2 + 3x + 2) = 5(x + 1)(x + 2)$

Factoring $ax^2 + bx + c$

Factor.

c) $2x^2 + 7x + 6$

d) $4x^2 – 7x + 3$

e) $3x^2 – 7x – 6$

Factoring $ax^2 + bx + c$

Solutions

c) $(2x + 3)(x + 2)$

d) $(4x – 3)(x – 1)$

e) $(3x + 2)(x – 3)$

Factoring $ax^2 + bx + c$

Factor.

f) $-4x^2 - 8x + 5$

g) $-9x^2 – 3x + 2$

Factoring $ax^2 + bx + c$

Solutions

f) $-(4x^2 + 8x - 5) = -(2x + 5)(2x - 1)$

g) $-(9x^2 + 3x - 2) = -(3x + 2)(3x - 1)$

Factoring Special Products

I can factor the difference of 2 squares and factor perfect square trinomials.

Factoring Special Products

Core Concepts (page 236 in Student Journal)

Difference of 2 Squares

$a^2 - b^2 = (a + b)(a - b)$

Perfect Square Trinomial

$a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$

Factoring Special Products

Examples (space on page 236 in Student Journal)

Factor.

a) $x^2 - 64 = x^2 - 8^2 = (x - 8)(x + 8)$

b) $25b^2 - 36 = (5b)^2 - 6^2 = 5b + 6)(5b - 6)$

Factoring Special Products

Solutions

a) $(x + 8)(x – 8)$

b) $(5b + 6)(5b – 6)$

Factoring Special Products

Factor.

c) $x^2 + 26x + 169, = x^2 + 26x + 13^2 = x^2 + 2.13.x + 13^2 = (x + 13)^2$

d) $9x^2 - 24x + 16 = (3x)^2 - 24x + 4^2 = (3x)^2 - 2.4.3x + 4^2 = (x - 4)^2$

Factoring Special Products

Solutions

c) $(x + 13)^2$

d) $(3x – 4)^2$

Factoring Polynomials Completely

I can factor by grouping and factoring completely.

Factoring Polynomials Completely

Vocabulary

factor by grouping: a strategy for factoring a polynomial with 4 terms by grouping the terms into pairs and factoring out the GCF from both pairs

Factoring Polynomials Completely

factored completely: a polynomial written as a product of unfactorable polynomials

Factoring Polynomials Completely

Examples

Factor by grouping.

a) $x^3 + 4x^2 + 2x + 8 = (x^3 + 4x^2) + (2x + 8)$

$= x^2(x + 4) + 2(x + 4) = (x^2 + 2)(x + 4) = \left(x^2 + \sqrt{2}^2\right)(x + 4) = x = -4,\ x^2 = -\sqrt{2}^2 =$

Factoring Polynomials Completely

Solutions

b) b) $x^2 + 4y + 2x + 2xy =$

$(x^2 + 2x) + (4y + 2xy) =$

$x(x + 2) + 2y(2 + x) =$

$(x + 2y)(x + 2)$

Factoring Polynomials Completely

Factor completely.

c) $2x^3 + 6x^2 – 2x$

d) $5x^4 – 45x^2$

Factoring Polynomials Completely

Solutions

c) $2x(x^2 + 3x - 1)$

d) $5x^2(x^2 - 9) = 5x^2(x + 3)(x - 3)$

Factoring Polynomials Completely

Solve.

e) $3x^3 + 6x^2 = 24x$

f) $2x^3 + 8x^2 = 10x$

Factoring Polynomials Completely

Solutions

e) $3x^3 + 6x^2 - 24x = 0$, $3x(x^2 + 2x - 8) = 0$,
$3x(x + 4)(x - 2) = 0$, $x = -4, 0, 2$

f) $2x^3 + 8x^2 - 10x = 0$, $2x(x^2 + 4x - 5) = 0$,
$2x(x + 5)(x - 1) = 0$, $x = -5, 0, 1$