GREATEST COMMON FACTORS (GCF)

OR

HIGHEST COMMON FACTOR (HCF)

&

LEAST COMMON MULTIPLES (LCM)

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HCF

- Definition

- The largest number that divides exactly into two or more numbers

Finding the HCF

- Begin with two numbers. 30 and 84
- Step 1 – Complete the prime factorization of these numbers. Do not use exponents.

  2 x 3 x 5 =30

  2 x 2 x 3 x 7=84

Step 2 – Write the prime factorizations underneath each other

2 x 3 x 5=30

2 x 2 x 3 x 7=84

- Step 3 – Circle each number that appears in both factorizations

  2 x 3 x 5 = 30

  2 x 2 x 3 x 7 = 84

- Note-

The second 2 is not circled in the bottom prime factorization because 2 only appears once in the first factorization.

Step 4-Multiply the circled numbers. 2 x 3=6

6 IS YOUR GCF FOR 30 AND 84

Find the HCF of 24 and 96

- Step 1-Prime Factorization.

  $2\ x\ 2\ x\ 2\ x\ 3 = 24\ \&\ 2\ x\ 2\ x\ 2\ x\ 2\ x\ 2\ x\ 3 = 96$

- Step 2-Write the prime factorizations underneath each other.

2 x 2 x 2 x 3=24

2 x 2 x 2 x 2 x 2 x 3=96

Step 3-Circle the common numbers.

2 x 2 x 2 x 3 = 24

2 x 2 x 2 x 2 x 2 x 3 = 96

Step 4-Multiply the circled numbers.

2 x 2 x 2 x 3=24

24 IS YOUR HCF FOR 24 AND 96

Find the GCF of 18 and 35

- Step 1-Prime Factorization.

2 x 3 x 3=18 35 5 x 7=35

- Step 2-Write the prime factorizations underneath each other.

2 x 3 x 3=18

5 x 7=35

- Step 3-Circle the common numbers.
- ***WE DON'T HAVE ANY COMMON NUMBERS!***
- ***Note-***
- ***If the prime factorizations have no numbers in common, the GCF is 1***.
- ***1 IS YOUR GCF FOR 18 AND 35***

Find the GCF of 15 and 36

Step 1-Prime Factorization.

3 x 5=15 & 2 x 2 x 3 x 3=36

Step 2-Write the prime factorizations underneath each other.

3 x 5=15

2 x 2 x 3 x 3=36

Step 3-Circle the common numbers.

3 x 5=15

2 x 2 x 3 x 3=36

Note-

If the prime factorizations only have one number in common, that number is your GCF.

3 IS YOUR GCF FOR 15 AND 36

- To find the GCF for more than 2 numbers,
- Take the same steps as before, but you will circle the common numbers in all of the prime factorizations

Begin with the given numbers. 24, 48, and 60

Step 1 – Complete the prime factorization of these numbers. Do not use exponents.

2 x 2 x 2 x 2 x 3=48 , 2 x 2 x 2 x 3 =24, 2 x 2 x 3 x 5=60

Step 2 – Write the prime factorizations underneath each other

2 x 2 x 2 x 3 = 24

2 x 2 x 2 x 2 x 3 = 48

2 x 2 x 3 x 5 = 60

Step 3 – Circle each number that appears in all factorizations

2 x 2 x 2 x 3 = 24

2 x 2 x 2 x 2 x 3 = 48

2 x 2 x 3 x 5 = 60

Step 4-Multiply the circled numbers. 2 x 2 x 3=12

12 IS YOUR GCF FOR 24, 48, AND 60

- NOTE-IF NO NUMBER APPEARS IN ALL PRIME FACTORIZATIONS, THEN GCF IS 1.

DIVISOR METHOD TO FIND HCF

- Divide the greater number by the smaller number,
- divide the divisor by the remainder,
- divide the remainder by the next remainder,
- and so on until no remainder is left.
- The last divisor is the required HCF.

Find highest common factor (H.C.F) of 18 and 30 by using division method.

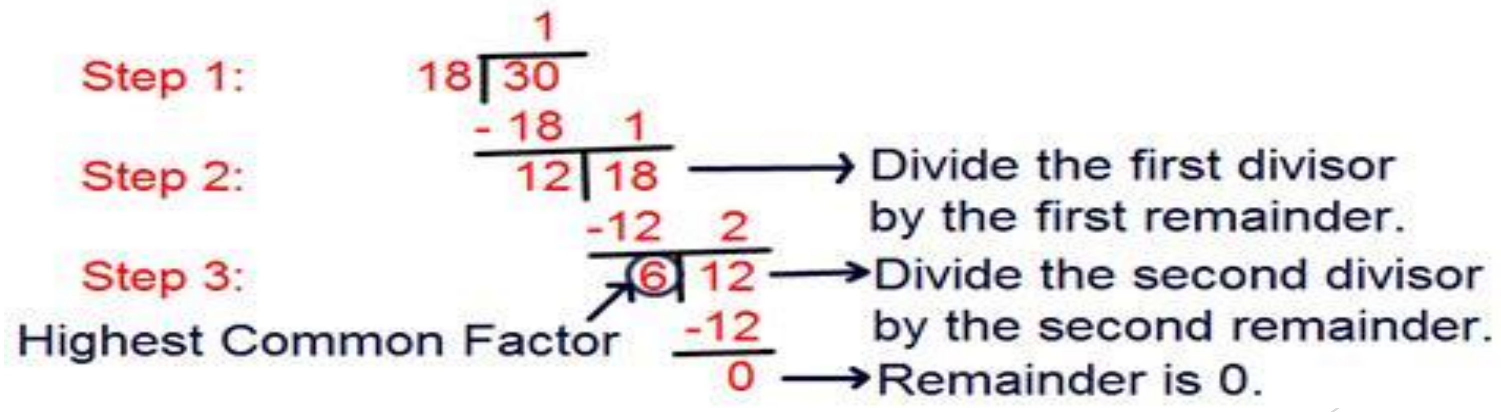
Step I: Here we need to divide 30 by 18. [Divide the larger number by the smaller one].

Step II: The first divisor is 18 and the remainder is 12, so we need to divide 18 by 12. [Divide the first divisor by the first remainder].

Step III: Now divide the second divisor 12 by the second remainder 6.
[Divide the second divisor by the second remainder].
Step IV: The remainder becomes 0.

Step V: Therefore, **highest common factor = 6**.

[The last divisor is the required highest common factor (H.C.F) of the given numbers].



Find highest common factor (H.C.F) of 75 and 180 by using division method.

SOLUTION:

Step I: Here we need to divide 180 by 75.
[Divide the larger number by the smaller one].

Step II: The first divisor is 75 and the remainder is 30,

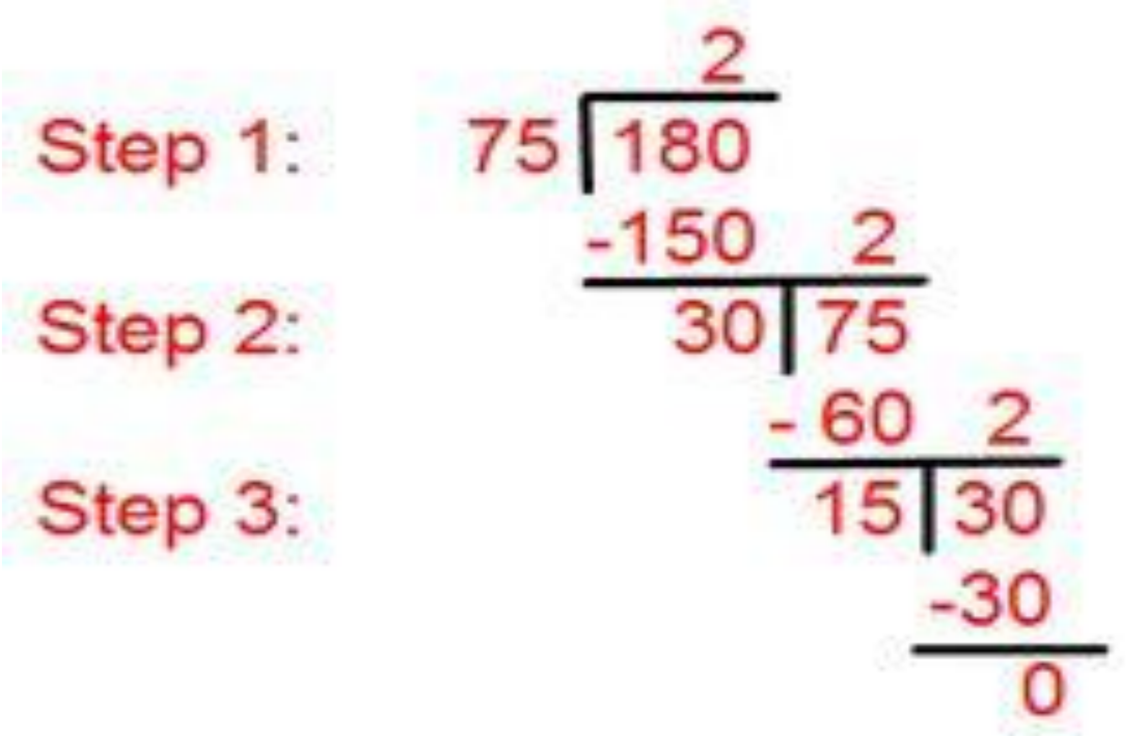
so we need to divide 75 by 30.
[Divide the first divisor by the first remainder].

Step III: Now divide the second divisor 30 by

the second remainder 15.
[Divide the second divisor by the second remainder].

Step IV: The remainder becomes 0.

Step V:



THEREFORE, HIGHEST COMMON FACTOR = 15.

To Find the Highest Common Factor of Three Numbers by using Division Method

STEPS

- **Step I:**

  First of all find the highest common factor (H.C.F) of any two of the given numbers.

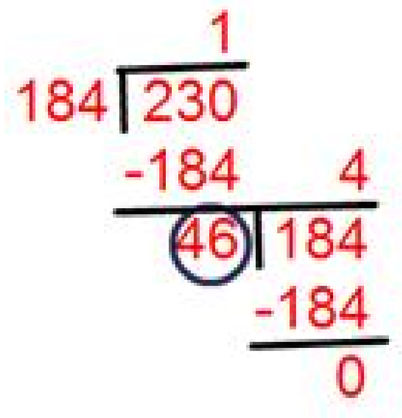
  Step II:

  Now find the highest common factor (H.C.F) of the third given number and the highest common factor (H.C.F) obtained in Step 1 from first and the second number

Find highest common factor (H.C.F) of 184, 230 and 276 by using division method.

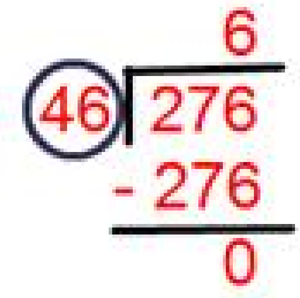
Solution:

Let us find the highest common factor (H.C.F) of 184 and 230.



Highest common factor of 184 and 230 = 46.

Now find the H.C.F. of 276 and 46.



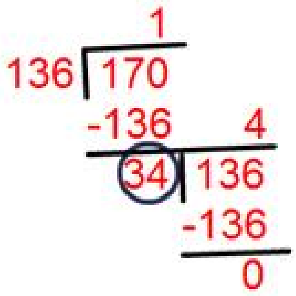
Highest common factor of 276 and 46 = 46.

Therefore, required highest common factor (H.C.F) of 184, 230 and 276 = 46.

Find highest common factor (H.C.F) of 136, 170 and 255 by using division method.

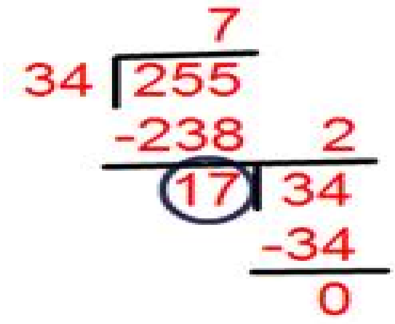
- **Solution:**

Let us find the highest common factor (H.C.F) of 136 and 170.



- Highest common factor of 136 and 170 = 34.

Now find the H.C.F. of 34 and 255.



- Highest common factor of 34 and 255 = 17.

THEREFORE, REQUIRED HIGHEST COMMON FACTOR (H.C.F) OF 136, 170 AND 255 = 17.

Using long division method, find H.C.F of 891, 1215 and 1377.

- Solution
- Consider 891 and 1215

```
        1
891 | 1215
     - 891       2
       324 | 891
           - 648      1
             243 | 324
                 - 243      3
                    81 | 243
                       - 243
                           0
```

- Therefore, highest common factor of 891 and 1215 is 81 and now we shall find H.C.F of 81 and 1377.

```
        17
81 | 1377
     - 81
      567
    - 567
        0
```

THEREFORE, H.C.F. OF 891, 1215 AND 1377 IS 81.

Least Common Multiple (LCM)

The smallest number that is a multiple of two or more numbers

FINDING THE LCM-ADVANCED METHOD

Begin with two numbers.

12 and 80

Step 1 – Complete the prime factorization of these numbers. Do not use exponents.

12	80
↙ ↘	↙ ↘
2 x 2 x 3 =12	2 x 2 x 2 x 2 x 5=80

FINDING THE LCM-ADVANCED METHOD

Step 2 – Write the prime factorizations underneath each other

$$2 \times 2 \times 3 = 12$$
$$2 \times 2 \times 2 \times 2 \times 5 = 80$$

Step 3 – Count the times each factor appears in each factorization.

$2 \times 2 \times 3 = 12$ ← 2 appears twice, 3 appears once

$2 \times 2 \times 2 \times 2 \times 5 = 80$ ← 2 appears four times, 5 appears once

FINDING THE LCM-ADVANCED METHOD

Step 4-Use the highest occurrence of each number to create a multiplication problem.

2 x 2 x 3=12 ← 2 appears twice, 3 appears once

2 x 2 x 2 x 2 x 5= 80 ← 2 appears four times, 5 appears once

2 x 2 x 2 x 2 x 3 x 5=

2 appears four times in the second factorization

3 appears once in the first factorization

5 appears once in the second factorization

LCM (Least Common Multiple)(continued):

1. Factorisation method:

Find all the prime factors of the given numbers.

Express the given numbers as the product of the prime factors.

The product of the highest powers of all factors is the LCM of the given numbers

Example:

1) Find the LCM of 45 and 75

Prime factors of 45 = 3 * 3 * 5 = 3^2 * 5

Prime factors of 75 = 3 * 5 * 5 = 3 * 5^2

Thus the LCM of 45 and 75 is 3^2 * 5^2 = 9 * 25 = 225

LCM (Least Common Multiple)(continued):

2. Division method:

Arrange the given numbers in a row.

Divide the given numbers by a number which exactly divides at least two of them and the numbers which are not divisible are carried forward.

The above process is repeated until no two of the given numbers are divisible by the same number except 1.

The product of all the divisors and the undivided numbers gives the LCM of the given numbers.

LCM (Least Common Multiple)(continued):

Example:

1) Find the LCM of 20, 35, 46 and 50.

2	20	35	48	50
2	10	35	24	25
5	5	35	12	25
	1	7	12	5

Required LCM = 2 * 2 * 5 * 7 * 12 * 5 = 8400

HCF and LCM of fractions:

$$\text{HCF} = \frac{\text{HCF of numerators}}{\text{LCM of denominators}}$$

$$\text{LCM} = \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$$

HCF and LCM of fractions(continued):

<u>**Example:**</u>

1) Find the HCF and LCM of $\frac{2}{3}$, $\frac{5}{4}$, $\frac{1}{3}$, $\frac{5}{2}$ and $\frac{7}{5}$

Numerators are 2, 5, 1 and 7.

Denominators are 2, 3, 4 and 5.

$$\textbf{HCF} = \frac{\text{HCF of numerators}}{\text{LCM of denominators}} = \frac{\text{HCF of 2, 5, 1 and 7}}{\text{LCM of 2, 3, 4 and 5}} = \frac{1}{60}$$

$$\textbf{LCM} = \frac{\text{LCM of numerators}}{\text{HCF of denominators}} = \frac{\text{LCM of 2, 5, 1 and 7}}{\text{HCF of 2, 3, 4 and 5}} = \frac{70}{1} = 70$$

CO-RELATION BETWEEN HCF AND LCM

There is an interesting co-relation between H.C.F and L.C.M. of two numbers. The product of the H.C.F. and L.C.M. of any two numbers is always equal to the product of those two numbers. However the same is not applicable to three or more numbers.

HCF and LCM of fractions(continued):

Note:

HCF * LCM = product of numbers

Example:

The HCF of two numbers is 12 and their LCM is 72.
If one of the numbers is 24, find the other.

Solution:

Let the required number be x. then,

HCF * LCM = product of numbers

12 * 72 = 24 * x

$$x = \frac{12 \times 72}{24} = 36$$

Thus the other number is 36.

Example 1:
Find the greatest number that will divide 400, 435 and 541 leaving 9, 10 and 14 as remainders respectively.

Solution:

The required number would be HCF of (400-9), (435-10) and (541-14)

So the HCF (391,425,527)

$391 = 17 \times 23$

$425 = 5 \times 5 \times 17$

$527 = 17 \times 31$

HCF = 17

Therefore the required number is 17.

Example 2:

A, B and C start to jog around a circular stadium. They complete their rounds in 36 seconds, 48 seconds and 42 seconds respectively. After how many seconds will they be together at the starting point?

Solution:

The required time is the LCM of all their lap times. This is the earliest when all three will intersect at the same point.

Required time is the LCM (36,48,42)

LCM = $2 \times 2 \times 3 \times 3 \times 4 \times 7$

LCM = 1008

Therefore the required time is 1008 seconds

Highest common factor and lowest common multiple of two numbers are 18 and 1782 respectively. One number is 162, find the other.

We know, H.C.F. × L.C.M. = First number × Second number

then we get,

$18 \times 1782 = 162 \times$ Second number

$(18 \times 1782)/162 =$ Second number

Therefore, the second number = 198

The highest common factor and the lowest common multiple of two numbers are 825 and 25 respectively. If one of the two numbers is 275, find the other number.

We know, H.C.F. × L.C.M. = First number × Second number then we get,

$$825 \times 25 = 275 \times \text{Second number}$$

$$(825 \times 25)/ 275 = \text{Second number}$$

Therefore, the second number = 75

The greatest number of four digits which is divisible by 15, 25, 40 and 75 is:

Greatest number of 4-digits is 9999.

L.C.M. of 15, 25, 40 and 75 is 600.

On dividing 9999 by 600, the remainder is 399.

Required number (9999 - 399) = 9600

ASSIGNMENT QUESTIONS

- **I.** Find highest common factor of the following by complete factorisation:
- (i) 48, 56, 72
- (ii) 198, 360
- (iii) 102, 68, 136
- (iv) 1024, 576
- (v) 405, 783, 513

- **II.** Find the H.C.F. by long division method:
- (i) 84, 144
- (ii) 120, 168
- (iii) 430, 516, 817
- (iv) 632, 790, 869
- (v) 291, 582, 776
- (vi) 219, 1321, 2320, 8526

ASSIGNMENT QUESTIONS

III. Find lowest common multiple of the following numbers:

(i) 16, 24, 40

(ii) 40, 56, 60

(iii) 207, 138

(iv) 72, 96, 120

(v) 120, 150, 135

(vi) 102, 170, 136