# Calculus-1 (Bridge Course) 2020-2021
# Complementary Course for B.Sc Chemistry/Physics

## Limits of Function Values

Let $f(x)$ be defined on an open interval about $x_0$, *except possibly at $x_0$ itself.* If $f(x)$ gets arbitrarily close to $L$ (as close to $L$ as we like) for all $x$ sufficiently close to $x_0$, we say that $f$ approaches the **limit** $L$ as $x$ approaches $x_0$, and we write

$$\lim_{x \to x_0} f(x) = L,$$

which is read "the limit of $f(x)$ as $x$ approaches $x_0$ is $L$". Essentially, the definition says that the values of $f(x)$ are close to the number $L$ whenever $x$ is close to $x_0$ (on either side of $x_0$).

How does the function

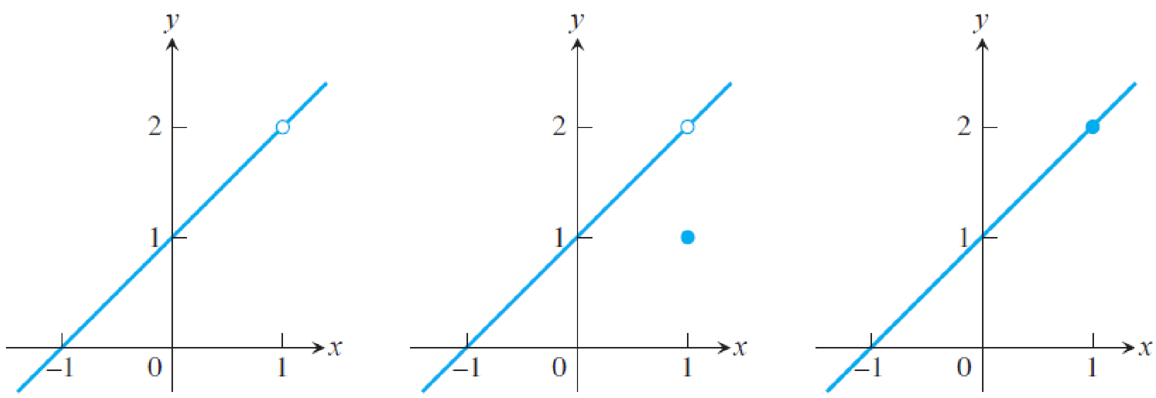
$$f(x) = \frac{x^2 - 1}{x - 1}$$

behave near $x = 1$?

**Solution** The given formula defines $f$ for all real numbers $x$ except $x = 1$ (we cannot divide by zero). For any $x \neq 1$, we can simplify the formula by factoring the numerator and canceling common factors:

$$f(x) = \frac{(x-1)(x+1)}{x-1} = x + 1 \qquad \text{for} \qquad x \neq 1.$$

## The Limit Value Does Not Depend on How the Function Is Defined at $x_0$




(a) $f(x) = \dfrac{x^2 - 1}{x - 1}$ (b) $g(x) = \begin{cases} \dfrac{x^2 - 1}{x - 1}, & x \neq 1 \\ 1, & x = 1 \end{cases}$ (c) $h(x) = x + 1$

The limits of $f(x)$, $g(x)$, and $h(x)$ all equal 2 as $x$ approaches 1. However, only $h(x)$ has the same function value as its limit at $x = 1$

Examples:

**(a)** $\lim_{x\to 2}(4) = 4$

**(b)** $\lim_{x\to -13}(4) = 4$

**(c)** $\lim_{x\to 3} x = 3$

**(d)** $\lim_{x\to 2}(5x - 3) = 10 - 3 = 7$

**(e)** $\lim_{x\to -2}\dfrac{3x + 4}{x + 5} = \dfrac{-6 + 4}{-2 + 5} = -\dfrac{2}{3}$

The Identity and Constant Functions Have Limits at Every Point

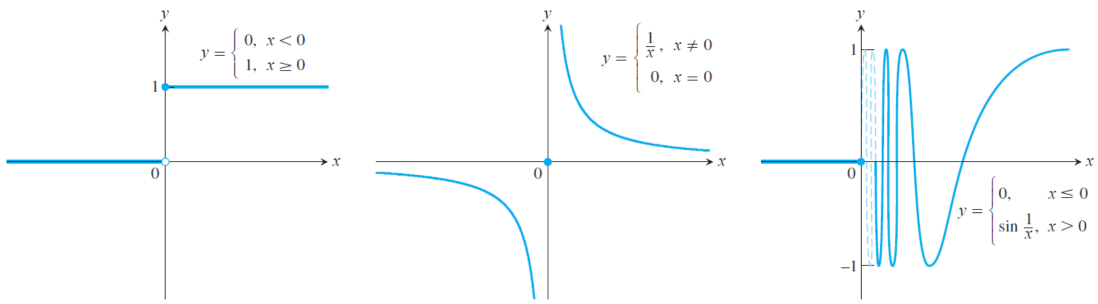
**(a)** If $f$ is the **identity function** $f(x) = x$, then for any value of $x_0$

$$\lim_{x\to x_0} f(x) = \lim_{x\to x_0} x = x_0.$$

**(b)** If $f$ is the **constant function** $f(x) = k$ (function with the constant value $k$), then for any value of $x_0$

$$\lim_{x\to x_0} f(x) = \lim_{x\to x_0} k = k.$$

Functions which do not posses limit at x=0.




(a) Unit step function $U(x)$ (b) $g(x)$ (c) $f(x)$

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## Exercise:

Which of the following statements about the function $y = f(x)$ graphed here are true, and which are false?

**a.** $\lim_{x\to 0} f(x)$ exists.

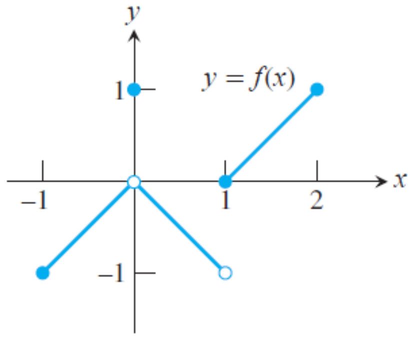
**b.** $\lim_{x\to 0} f(x) = 0$.

**c.** $\lim_{x\to 0} f(x) = 1$.

**d.** $\lim_{x\to 1} f(x) = 1$.

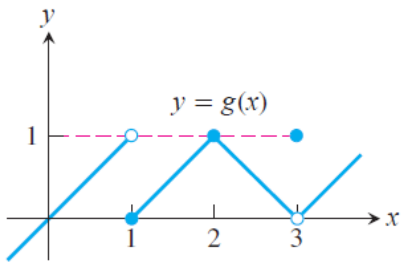
**e.** $\lim_{x\to 1} f(x) = 0$.

**f.** $\lim_{x\to x_0} f(x)$ exists at every point $x_0$ in $(-1, 1)$.



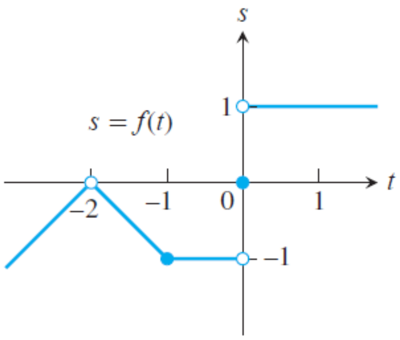

For the function $g(x)$ graphed here, find the following limits or explain why they do not exist.

**a.** $\lim_{x\to 1} g(x)$ **b.** $\lim_{x\to 2} g(x)$ **c.** $\lim_{x\to 3} g(x)$




For the function $f(t)$ graphed here, find the following limits or explain why they do not exist.

**a.** $\lim_{t\to -2} f(t)$ **b.** $\lim_{t\to -1} f(t)$ **c.** $\lim_{t\to 0} f(t)$



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Evaluate:

$\lim_{x \to 2} 2x$ $\qquad$ $\lim_{x \to 0} 2x$

$\lim_{x \to 1/3} (3x - 1)$ $\qquad$ $\lim_{x \to 1} \frac{-1}{(3x - 1)}$

$\lim_{x \to -1} 3x(2x - 1)$ $\qquad$ $\lim_{x \to -1} \frac{3x^2}{2x - 1}$

$\lim_{x \to \pi/2} x \sin x$ $\qquad$ $\lim_{x \to \pi} \frac{\cos x}{1 - \pi}$

**Exercise:**

Let $f(x) = (3^x - 1)/x$.

**a.** Make tables of values of $f$ at values of $x$ that approach $x_0 = 0$ from above and below. Does $f$ appear to have a limit as $x \to 0$? If so, what is it? If not, why not?

**b.** Support your conclusions in part (a) by graphing $f$ near $x_0 = 0$.

**Exercise:**

Let $g(x) = (x^2 - 2)/(x - \sqrt{2})$.

**a.** Make a table of the values of $g$ at the points $x = 1.4, 1.41, 1.414$, and so on through successive decimal approximations of $\sqrt{2}$. Estimate $\lim_{x \to \sqrt{2}} g(x)$.

**b.** Support your conclusion in part (a) by graphing $g$ near $x_0 = \sqrt{2}$ and using Zoom and Trace to estimate $y$-values on the graph as $x \to \sqrt{2}$.

**c.** Find $\lim_{x \to \sqrt{2}} g(x)$ algebraically.