Correlation and Auto-Correlation of Signals

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Objectives

- Develop an intuitive understanding of the cross-correlation of two signals.
- Define the meaning of the auto-correlation of a signal.
- Develop a method to calculate the cross-correlation and auto-correlation of signals.
- Demonstrate the relationship between auto-correlation and signal power.
- Demonstrate how to detect periodicities in noisy signals using auto-correlation techniques.
- Demonstrate the application of cross-correlation to sonar or radar ranging

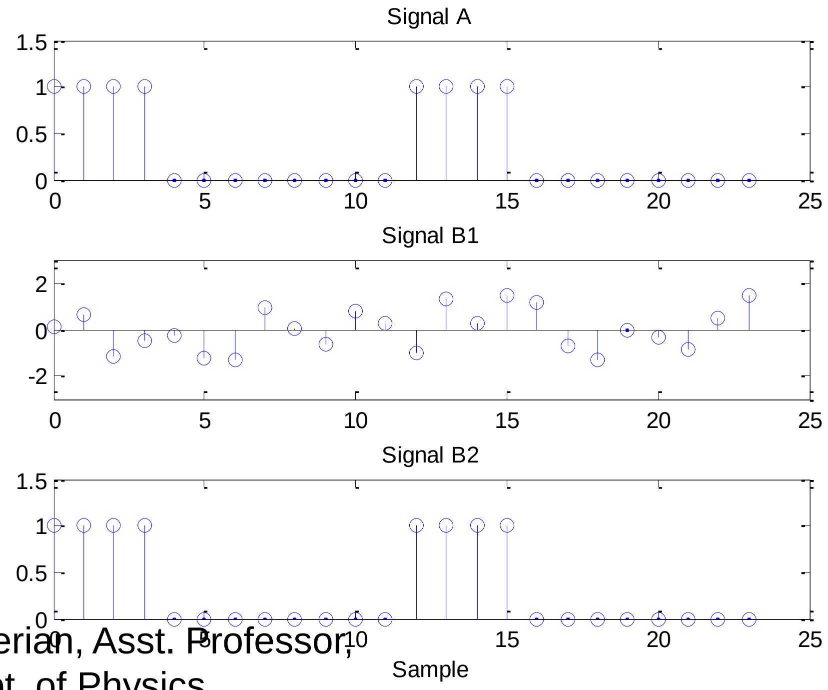
Correlation

- Correlation addresses the question: “to what degree is signal A similar to signal B.”
- An intuitive answer can be developed by comparing deterministic signals with stochastic signals.
  - Deterministic = a predictable signal equivalent to that produced by a mathematical function
  - Stochastic = an unpredictable signal equivalent to that produced by a random process

Three Signals

```
>> n=0:23;
>> A=[ones(1,4),zeros(1,8),ones(1,4),zeros(1,8)];
>> subplot (3,1,1),stem(n,A);axis([0 25 0 1.5]);title('Signal A')
>> B1=randn(size(A));  %The signal B1 is Gaussian noise with the same length as A
>> subplot(3,1,2),stem(n,B1);axis([0 25 -3 3]);title('Signal B1')
>> B2=A;
>> subplot(3,1,3),stem(n,B2); axis([0 25 0 1.5]);title('Signal B2');xlabel('Sample')
```

By inspection, A is "correlated" with B2, but B1 is "uncorrelated" with both A and B2. This is an intuitive and visual definition of "correlation."



Quantitative Correlation

- We seek a quantitative and algorithmic way of assessing correlation
- A possibility is to multiple signals sample-by-sample and average the results. This would give a relatively large positive value for identical signals and a near zero value for two random signals.

$$r_{12} = \frac{1}{N}\sum_{n=0}^{N-1} x_1[n]x_2[n]$$

Simple Cross-Correlation

- Taking the previous signals, A, B1(random), and B2 (identical to A):

```
>> A*B1'/length(A)
ans =
   -0.0047
>> A*B2'/length(A)
ans =
    0.3333
```

The small numerical result with A and B1 suggests those signals are uncorrelated while A and B2 are correlated.

Simple Cross-Correlation of Random Signals

```
>> n=0:100;
>> noise1=randn(size(n));
>> noise2=randn(size(n));
>> noise1*noise2'/length(noise1)
ans =
   0.0893
```

Are the two signals correlated?

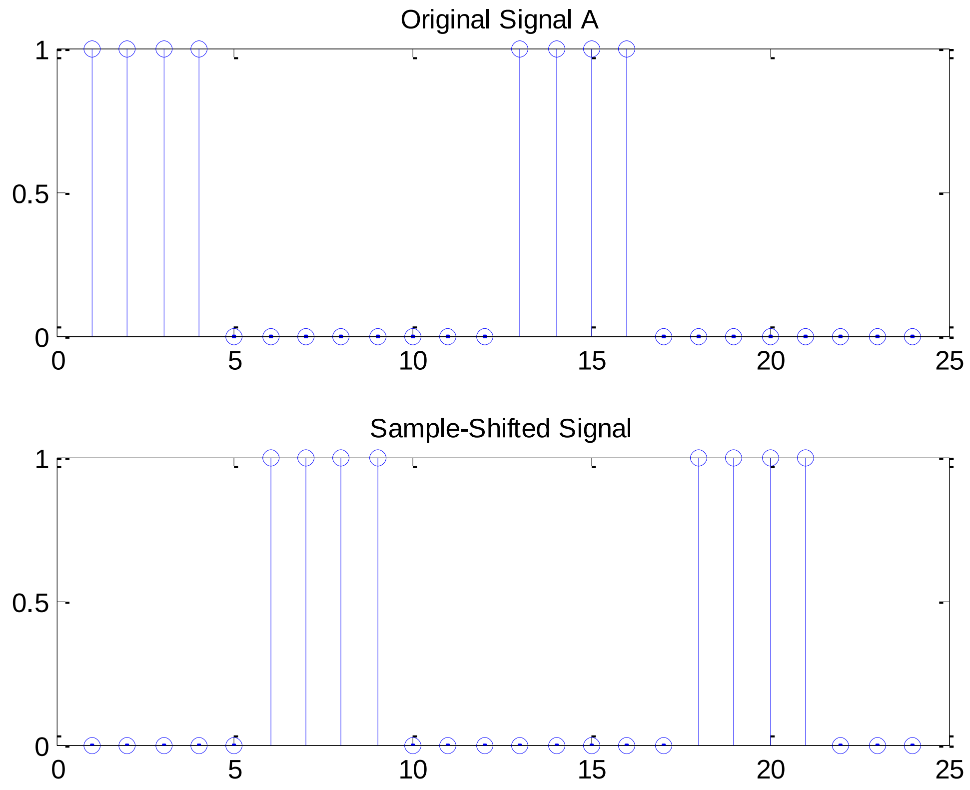
With high probability, the result is expected to be

$\leq \pm 2/\sqrt{N} = \pm 0.1990$

for two random (uncorrelated) signals

We would conclude these two signals are ***uncorrelated***.

The Flaw in Simple Cross-Correlation



In this case, the simple cross-correlation would be zero despite the fact the two signals are obviously "correlated."

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Sample-Shifted Cross-Correlation

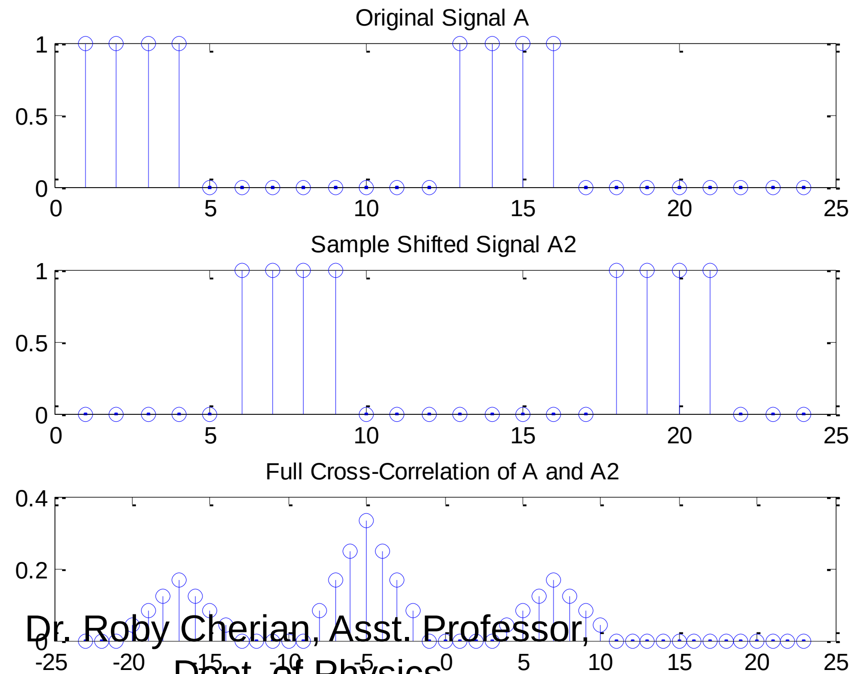
- Shift the signals k steps with respect to one another and calculate $r_{12}(k)$.
- All possible k shifts would produce a vector of values, the “full” cross-correlation.
- The process is performed in MATLAB by the command **xcorr**
- **xcorr** is equivalent to **conv** (convolution) with one of the signals taken in reverse order.

$$r_{12}(k) = \frac{1}{N}\sum_{n=0}^{N-1} x_1[n]x_2[n+k]$$

Full Cross-Correlation

```
>> A=[ones(1,4),zeros(1,8),ones(1,4),zeros(1,8)];
>> A2=filter([0,0,0,0,0,1],1,A);
>> [acor,lags]=xcorr(A,A2);
>> subplot(3,1,1),stem(A); title('Original Signal A')
>> subplot(3,1,2),stem(A2); title('Sample Shifted Signal A2')
>> subplot(3,1,3),stem(lags,acor/length(A)),title('Full Cross-Correlation of A and A2')
```

Signal A2 shifted to the left by 5 steps makes the signals identical and $r_{12} = 0.333$



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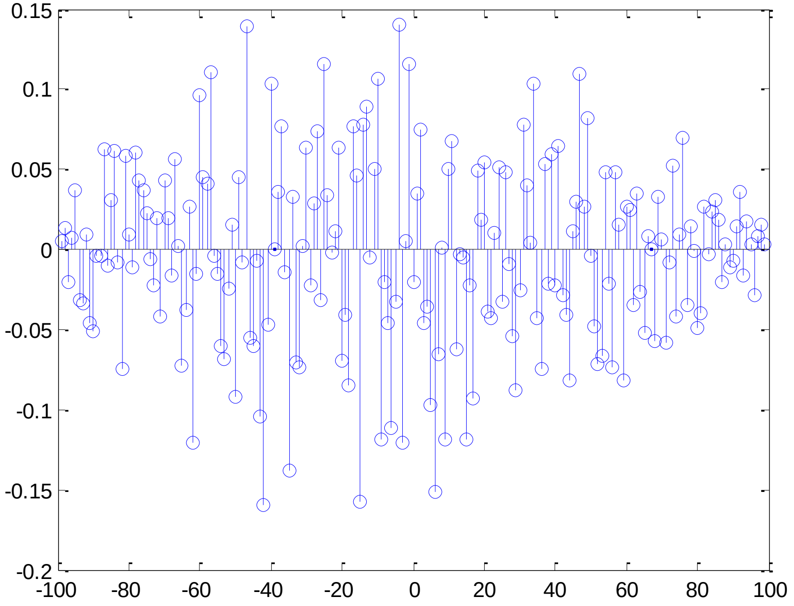
Full Cross-Correlation of Two Random Signals

```
>> N=1:100;
>> n1=randn(size(N));
>> n2=randn(size(N));
>> [acor,lags]=xcorr(n1,n2);
>> stem(lags,acor/length(n1));
```

The cross-correlation is random and shows no peak, which implies no correlation



Auto-Correlation

- The cross-correlation of a signal with itself is called the *auto-correlation*

$$r_{11}(k) = \frac{1}{N}\sum_{n=0}^{N-1} x_1[n]x_1[n+k]$$

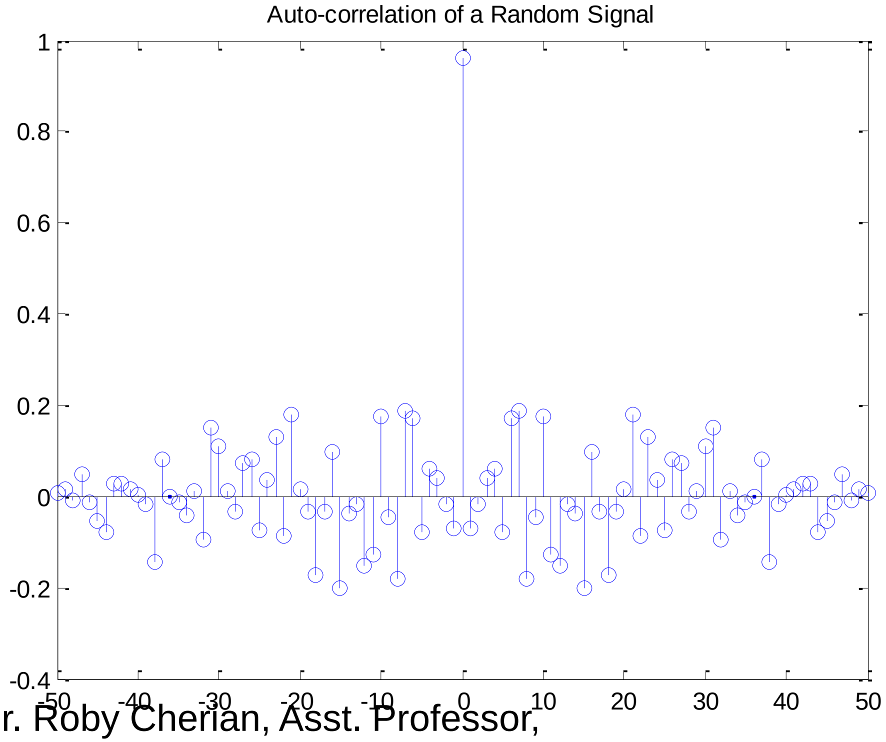
- The "zero-lag" auto-correlation is the same as the mean-square *signal power*.

$$r_{11}(0) = \frac{1}{N}\sum_{n=0}^{N-1} x_1[n]x_1[n] = \frac{1}{N}\sum_{n=0}^{N-1} x^2[n]$$

Auto-Correlation of a Random Signal

```
>> n=0:50;
>> N=randn(size(n));
>> [rNN,k]=xcorr(N,N);
>> stem(k,rNN/length(N));title('Auto-correlation of a Random Signal')
```

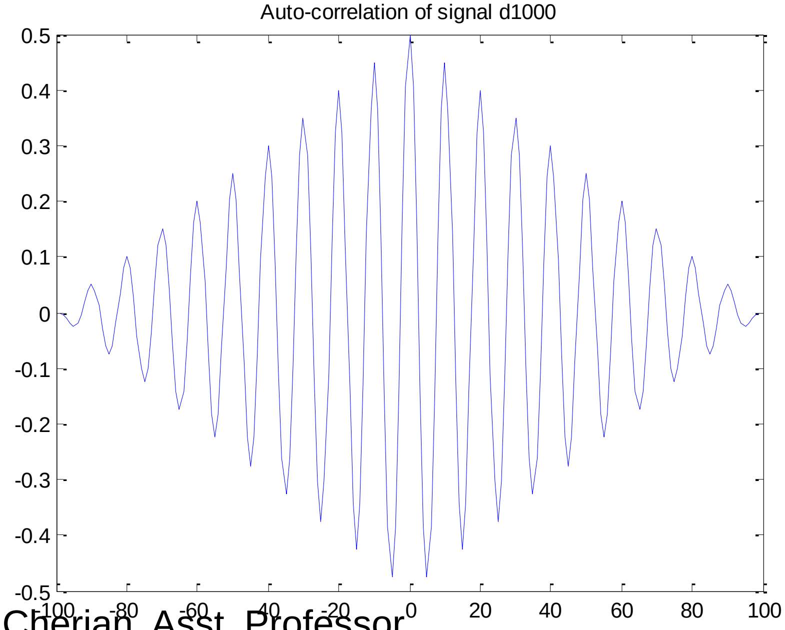
Mathematically, the auto-correlation of a random signal is like the impulse function



Auto-Correlation of a Sinusoid

```
>> n=0:99;
>> omega=2*pi*100/1000;
>> d1000=sin(omega*n);
>> [acor_d1000,k]=xcorr(d1000,d1000);
>> plot(k,acor_d1000/length(d1000));
>> title('Auto-correlation of signal d1000')
```

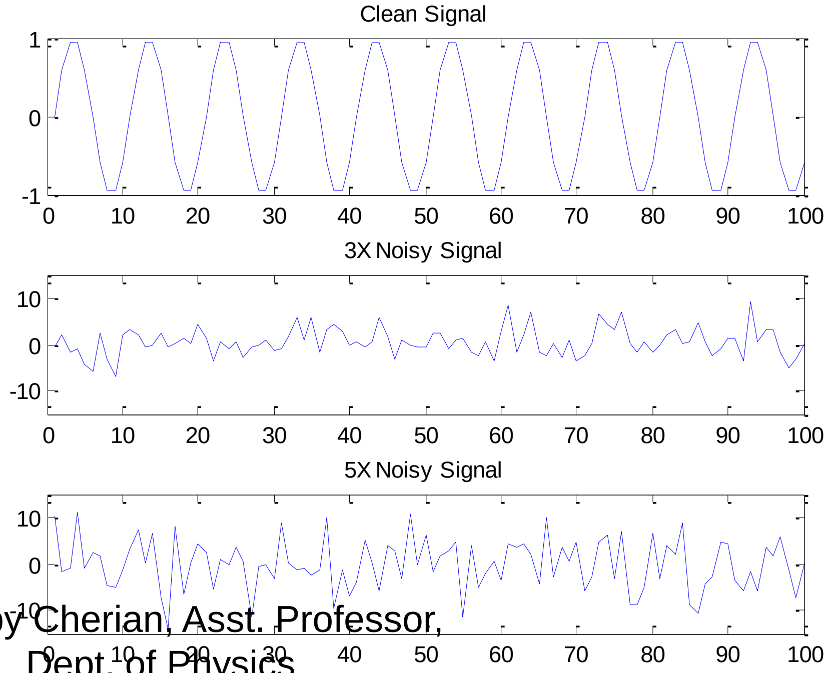
The auto-correlation vector has the same frequency components as the original signal



Identifying a Sinusoidal Signal Masked by Noise

```
>> n=0:1999;
>> omega=2*pi*100/1000;
>> d=sin(omega*n);
>> d3n=d+3*randn(size(d));  % The sinusoid is contaminated with 3X noise
>> d5n=d+5*randn(size(d));  % The sinusoid is contaminated with 5X noise.
>> subplot(3,1,1),plot(d(1:100)),title('Clean Signal')
>> subplot(3,1,2),plot(d3n(1:100)),title('3X Noisy Signal'), axis([0,100,-15,15])
>> subplot(3,1,3),plot(d5n(1:100)),title('5X Noisy Signal'), axis([0,100,-15,15])
```

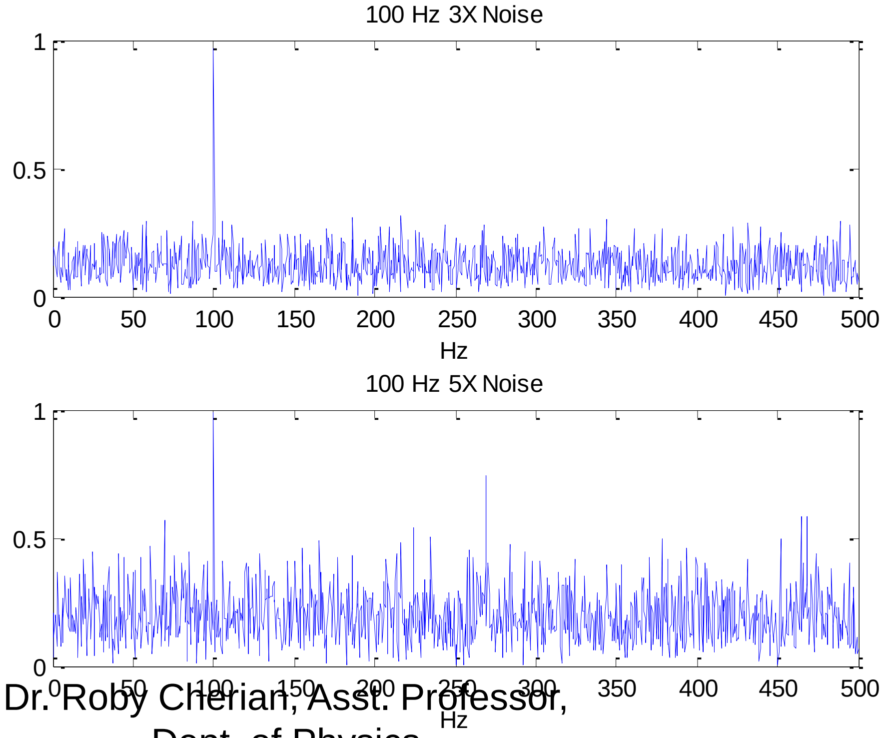
It is very difficult to “see” the sinusoid in the noisy signals



Identifying a Sinusoidal Signal Masked by Noise (Normal Spectra)

```
>> n=0:1999;
>> omega=2*pi*100/1000;
>> d=sin(omega*n);
>> d3n=d+3*randn(size(d));  % The sinusoid is contaminated with 3X noise
>> d5n=d+5*randn(size(d));  % The sinusoid is contaminated with 5X noise.
>> subplot(2,1,1),fft_plot(d3n,1000);title('100 Hz 3X Noise')
>> subplot(2,1,2),fft_plot(d5n,1000);title('100 Hz 5X Noise')
```

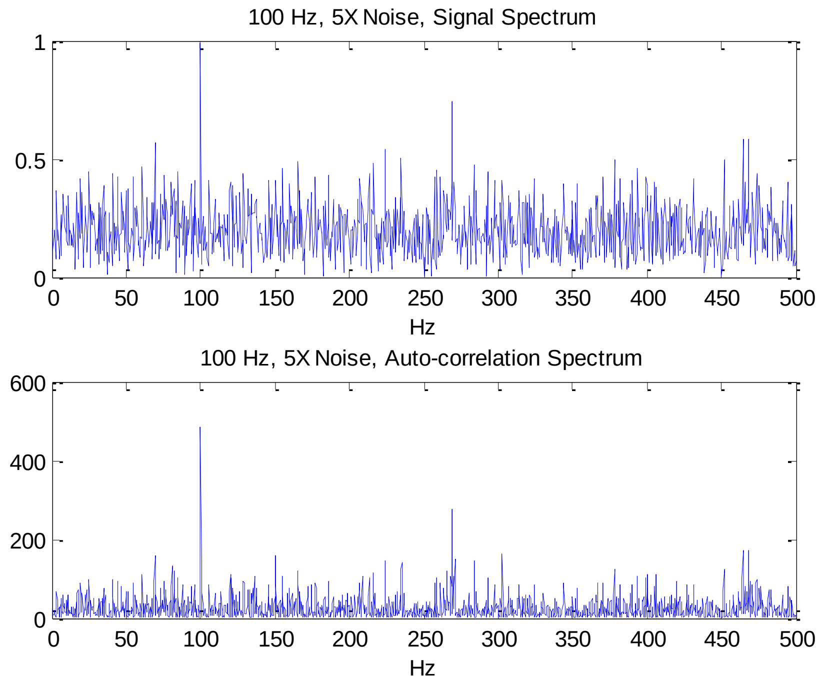
Normal spectra of a sinusoid masked by noise: High noise power makes detection less certain



Identifying a Sinusoidal Signal Masked by Noise (Auto-correlation Spectra)

```
>> acor3n=xcorr(d3n,d3n);
>> acor5n=xcorr(d5n,d5n);
>> subplot(2,1,1),fft_plot(d3n,1000);title('100 Hz, 3X Noise, Signal Spectrum')
>> subplot(2,1,2),fft_plot(acor3n,1000);title('100 Hz, 3X Noise, Auto-correlation Spectrum')
>> figure, subplot(2,1,1),fft_plot(d5n,1000);title('100 Hz, 5X Noise, Signal Spectrum')
>> subplot(2,1,2),fft_plot(acor5n,1000);title('100 Hz, 5X Noise, Auto-correlation Spectrum')
```

The ***<u>auto-correlation</u>*** of a noisy signal provides greater S/N in detecting dominant frequency components compared to a normal FFT



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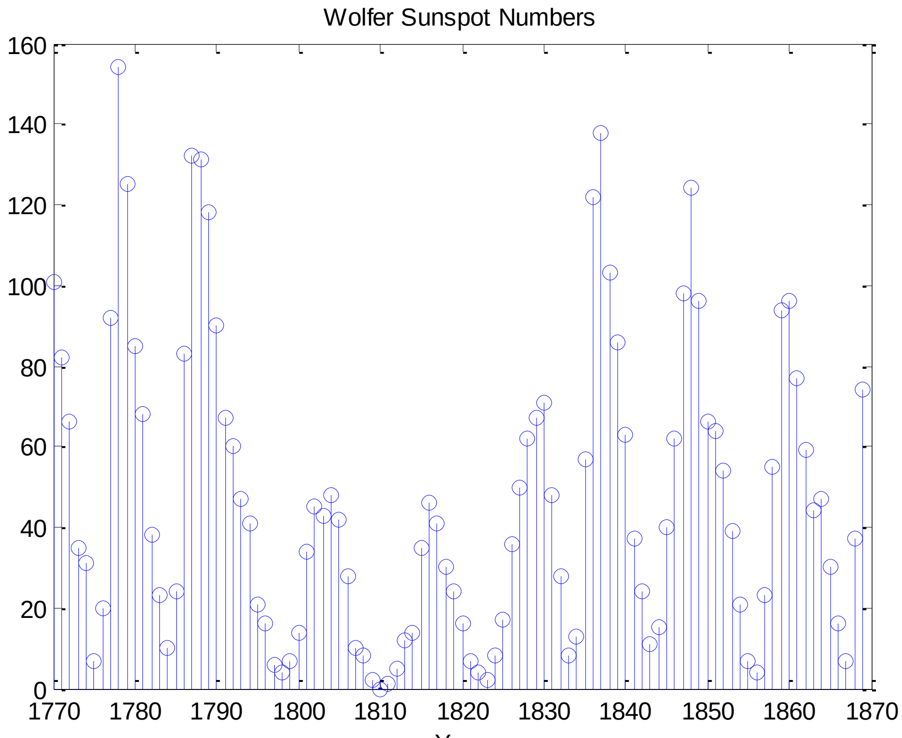
Detecting Periodicities in Noisy Data: Annual Sunspot Data

```
>> load wolfer_numbers
>> year=sunspots(:,1);
>> spots=sunspots(:,2);
>> stem(year,spots);title('Wolfer Sunspot Numbers');xlabel('Year')
```



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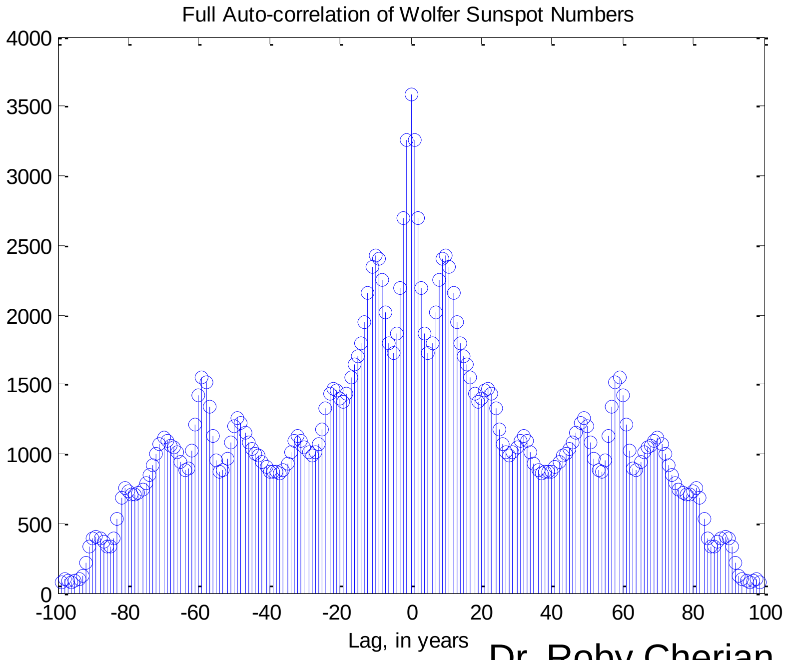
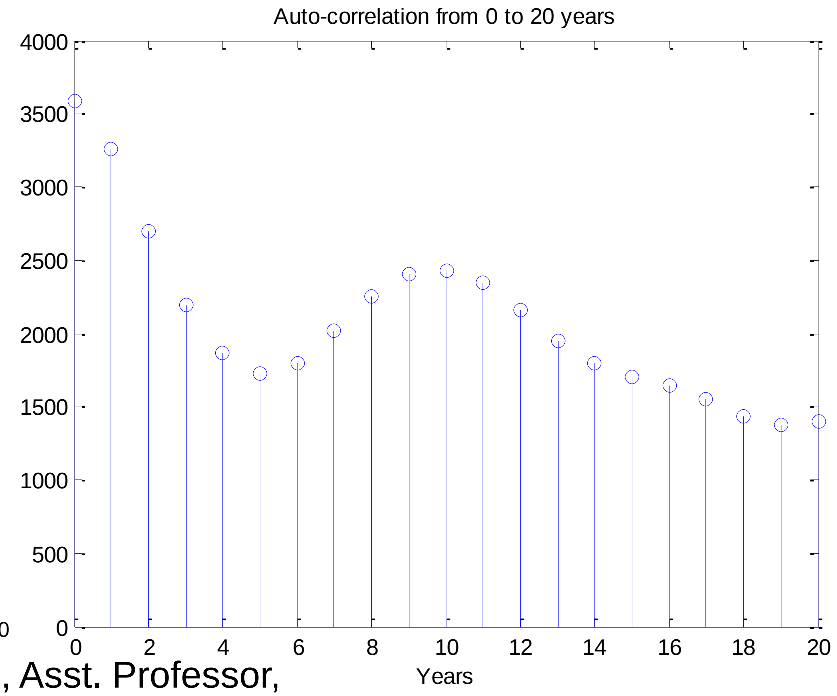
Detecting Periodicities in Noisy Data: Annual Sunspot Data

```
>> [acor,lag]=xcorr(spots);
>> stem(lag,acor/length(spots));
>> title('Full Auto-correlation of Wolfer Sunspot Numbers')
>> xlabel('Lag, in years')
>> figure, stem(lag(100:120),acor(100:120)/length(spots));
>> title('Auto-correlation from 0 to 20 years')
>> xlabel('Years')
```

Autocorrelation has detected a periodicity of 9 to 11 years

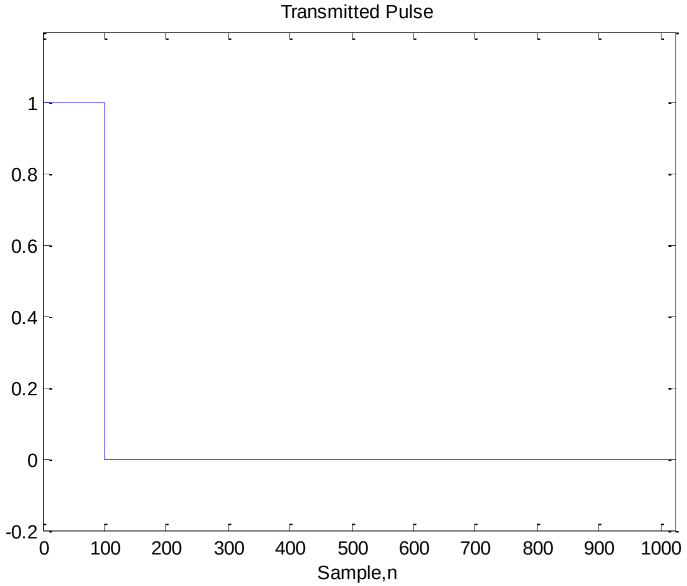


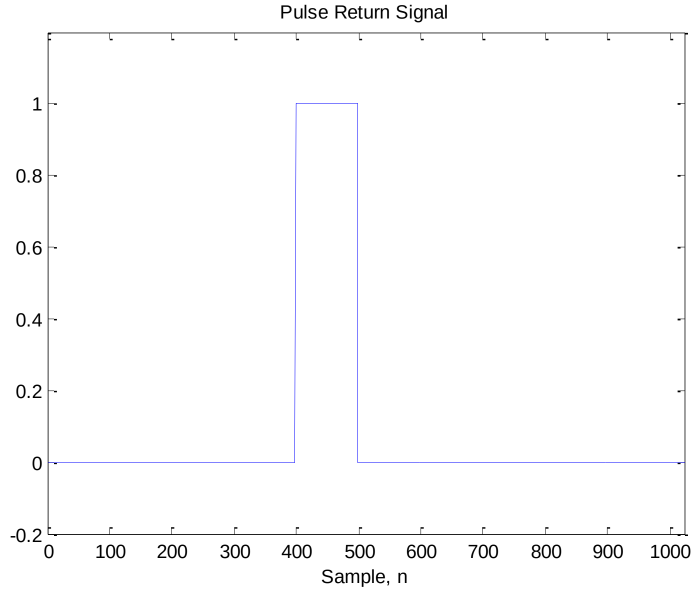


Sonar and Radar Ranging

```
>> x=[ones(1,100),zeros(1,924)];
>> n=0:1023;
>> plot(n,x); axis([0 1023 -.2, 1.2])
>> title('Transmitted Pulse');xlabel('Sample,n')
>> h=[zeros(1,399),1];            % Impulse response for z-400 delay
>> x_return=filter(h,1,x);        % Put signal thru delay filter
>> figure,plot(n,x_return); axis([0 1023 -.2, 1.2])
>> title('Pulse Return Signal');xlabel('Sample, n')
```

Simulation of a transmitted and received pulse (echo) with a 400 sample delay

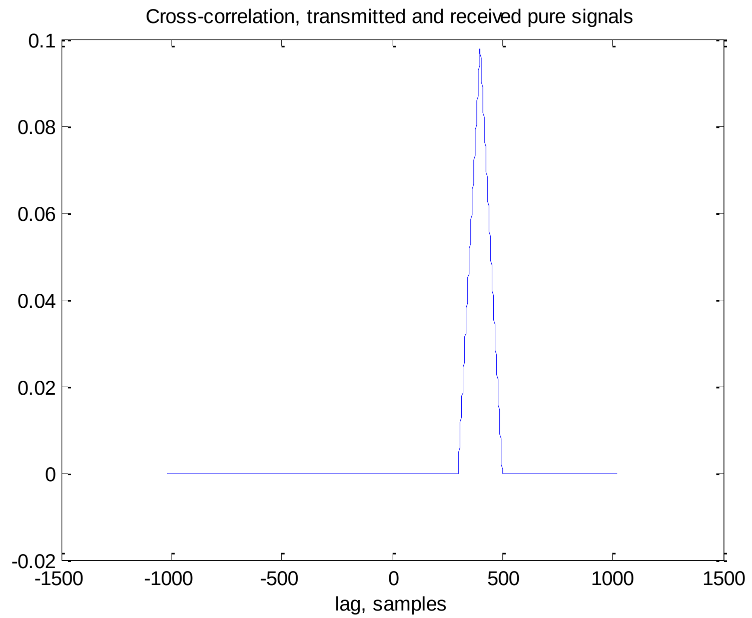




Sonar and Radar Ranging

```
>> [xcor_pure,lags]=xcorr(x_return,x);
>> plot(lags,xcor_pure/length(x))
>> title('Cross-correlation, transmitted and received pure signals')
>> xlabel('lag, samples')
```

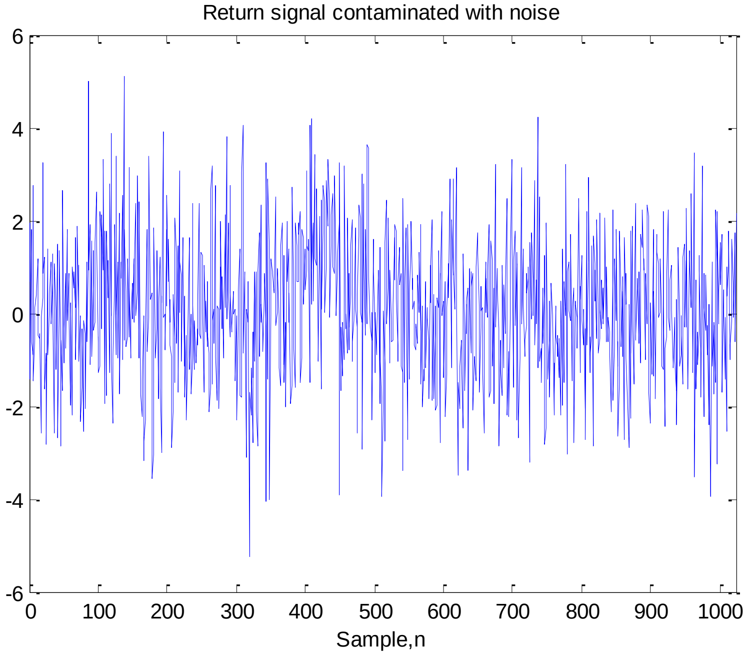
The cross-correlation of the transmitted and received signals shows they are correlated with a 400 sample delay



Sonar and Radar Ranging

```
>> x_ret_n=x_return+1.5*randn(size(x_return));
>> plot(n,x_ret_n); axis([0 1023 -6, 6])          %Note change in axis range
>> title('Return signal contaminated with noise')
>> xlabel('Sample,n')
```

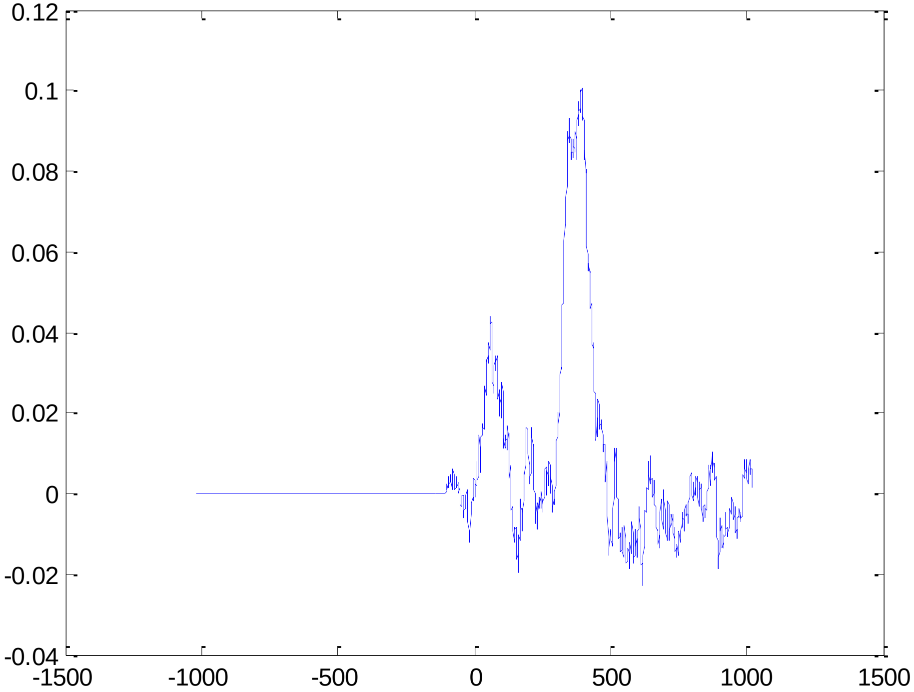
The presence of the return signal in the presence of noise is almost impossible to see



Sonar and Radar Ranging

```
>> [xcor,lags]=xcorr(x_ret_n,x);
>> plot(lags,xcor/length(x))
```

Cross-correlation of the transmitted signal with the noisy echo clearly shows a correlation at a delay of 400 samples



Summary

- Cross-correlation allows assessment of the degree of similarity between two signals.
  - Its application to identifying a sonar/radar return echo in heavy noise was illustrated.
- Auto-correlation (the correlation of a signal with itself) helps identify signal features buried in noise.