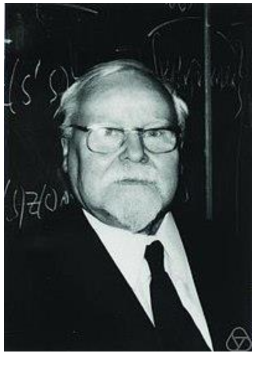
SEPARATION AXIOM

DISCLAIMER

Note that in this context the word axiom is not used in the meaning of "principle" of a theory, which has necessarily to be assumed, but in the meaning of "requirement" contained in a definition, which can be fulfilled or not, depending on the cases.



Notation

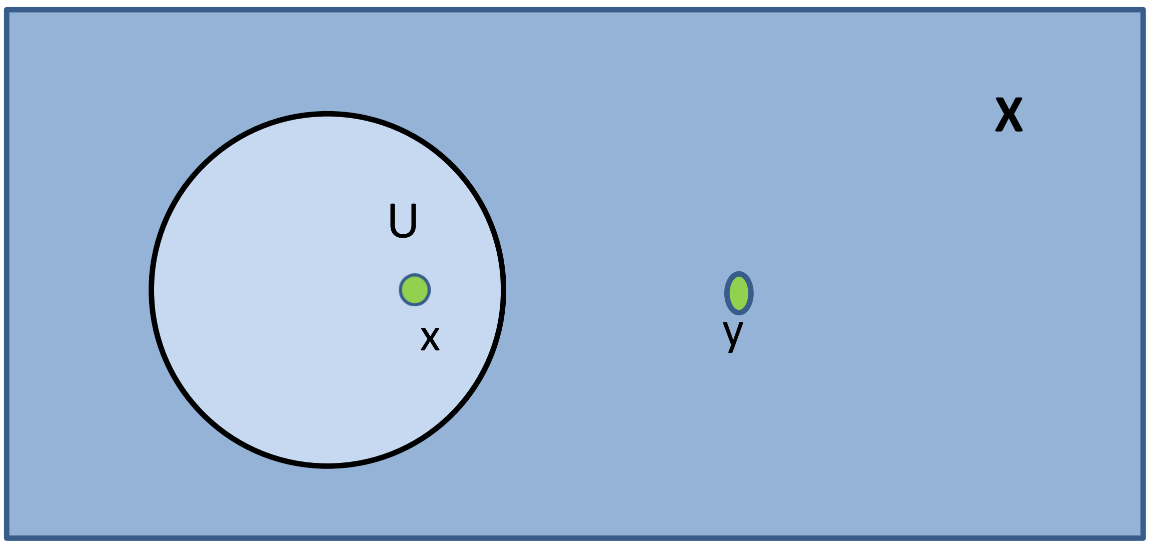
Circle – open sets

Rectangle with sharp edges – topological space

Rectangle with blunt edges – closed sets

T0-separation axiom
Kolmogorov space

For any two points x , y in X, there is an open set U such that x in U and y not in U or y in U and x not in U.

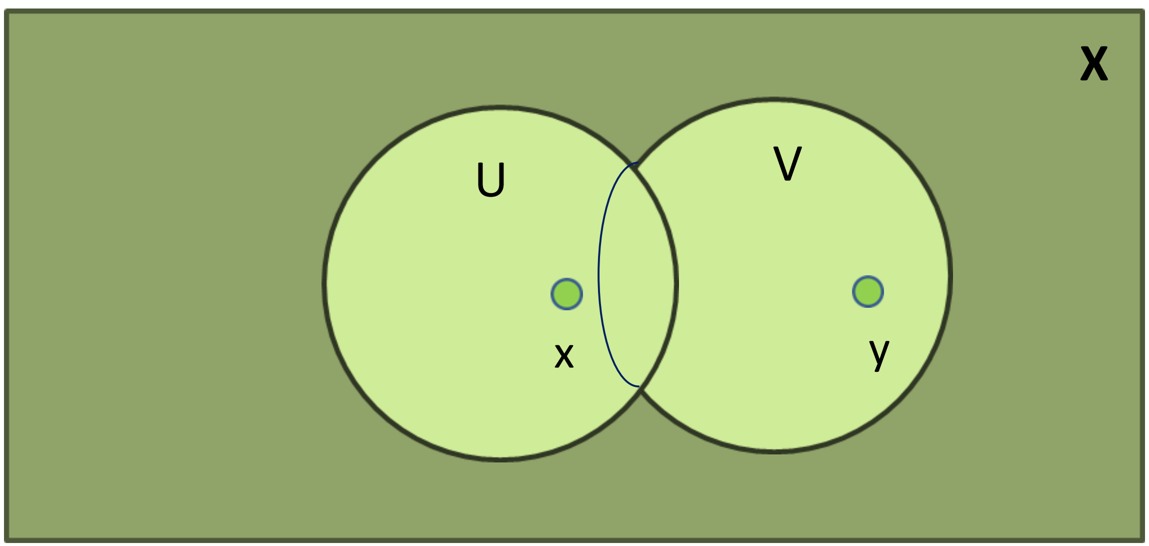


Example of To-space

- Every discrete topological space is To – space , for if x and y are two distinct points of X, then {x} contains x and does not contain y.
- A non-discrete space containing more than one point is not To – space.
- An indiscrete space is not To
- A cofinite topological space such that X is an infinite set for if x and y are two distinct points of X, then{x} being finite, X-{x} is open set containing x but not y.

T1-separation axiom
Frechet spaces

For any two points x, y in X there exists two open sets U and V such that x in U and y not in U, and y in V and x not in V.



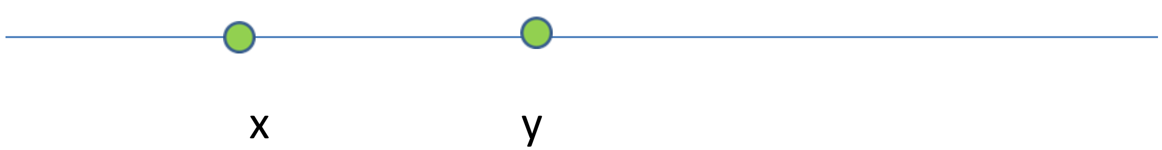
Example of T1-space

- Every discrete topological space with two or more points is T1 – space , for if x and y are two distinct points of X, then {x} contains x and does not contain y and {y} contains y that does not contain x.

Every T1 space is T0

- Let τ ={(a,∞), a in R}U{R,ϕ}, then (X, τ) is T0 space but not T1.

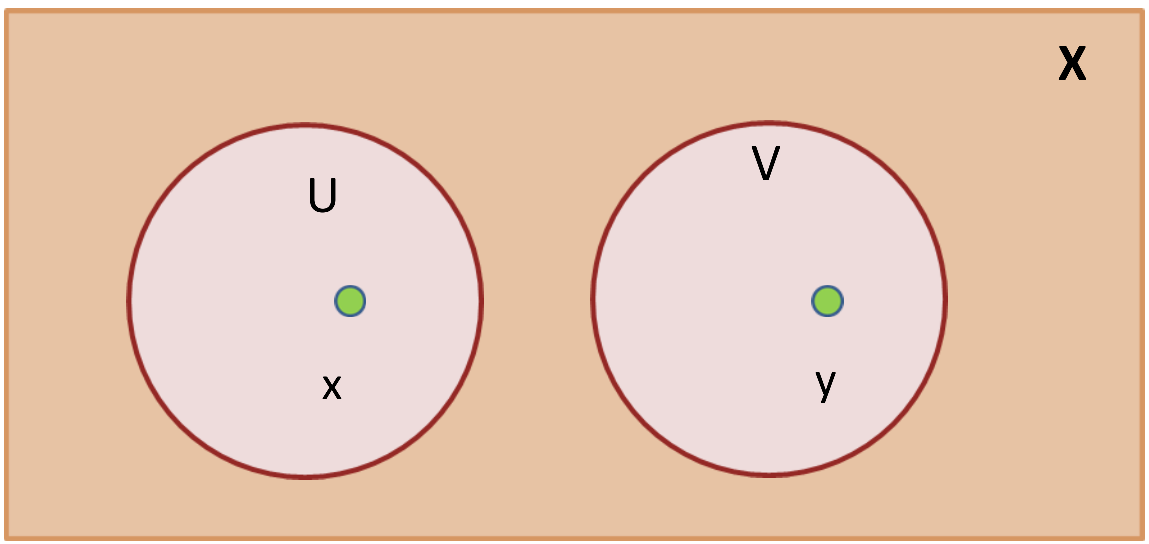
For let x,y ϵ R with x < y, then (x, ∞) contains y but there is no open set containing x but not y.



But R under usual topology is T1

T2-separation axiom
Huasdorff space

For any two points x,y in X there exists two open sets U and V such that x in U, y in V, and intersection of U and V is empty.



Example of T2 - space

- Every discrete topological space with two or more points is T2 – space , for if x and y are two distinct points of X, then {x} contains x and does not contain y and {y} contains y that does not contain x such that $\{x\} \cap \{y\} = \phi$

Every T2 space is T1

- Consider a cofinite topological space such that X is an infinite set

for if x and y are two distinct points of X, then{x} being finite, X-{x} is open set containing x but not y and X-{y} is an open set containing y but not x.

Therefore the space is T1

Conversly assume it is T2

- Then there exist open sets G and H such that

x is in G and y is in H such that $G \cap H = \phi$

Then $(G \cap H)^c = \phi^c$

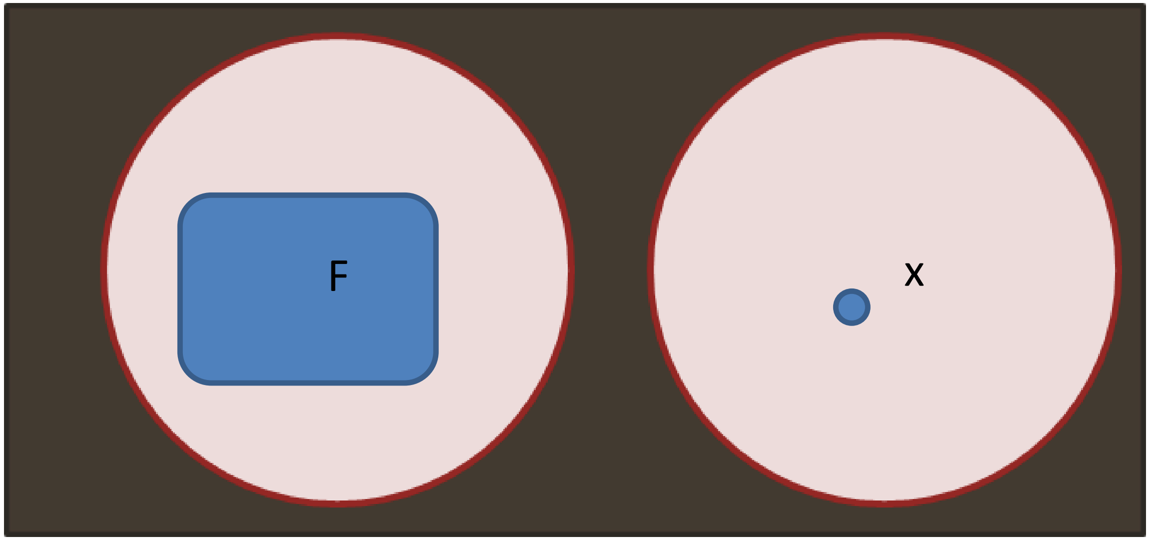
$G^c \cup H^c = X$

Since G and H are open its complements are finite and therefore it is a contradiction.

Hence the space is not T2

Regular

If, given any point x and closed set F in X such that x does not belong to F, there exist disjoint open sets U and V, such that U contains x and V contains F.



T3-separation axiom
Vietoris space

1.X is T3 if it fulfils T1 and is regular.

Let X = {a,b,c}

τ={X,φ, {b},{a,c}}

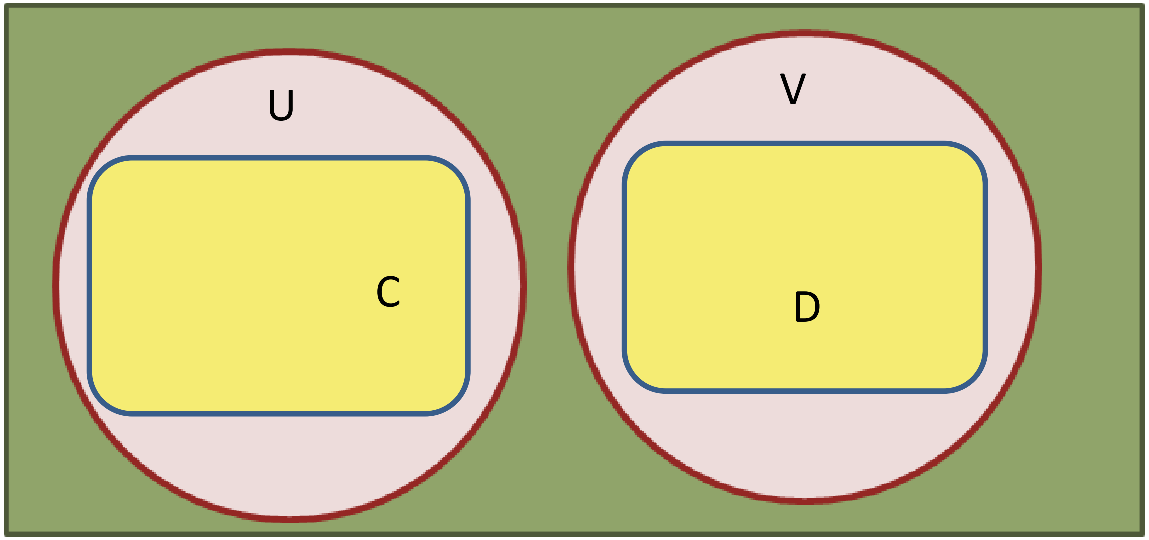
Then (X, τ) is regular but not T1(since there exist c and a), so it is not T3.

2. The usual topological space under R is T3

Every T3 space is T2

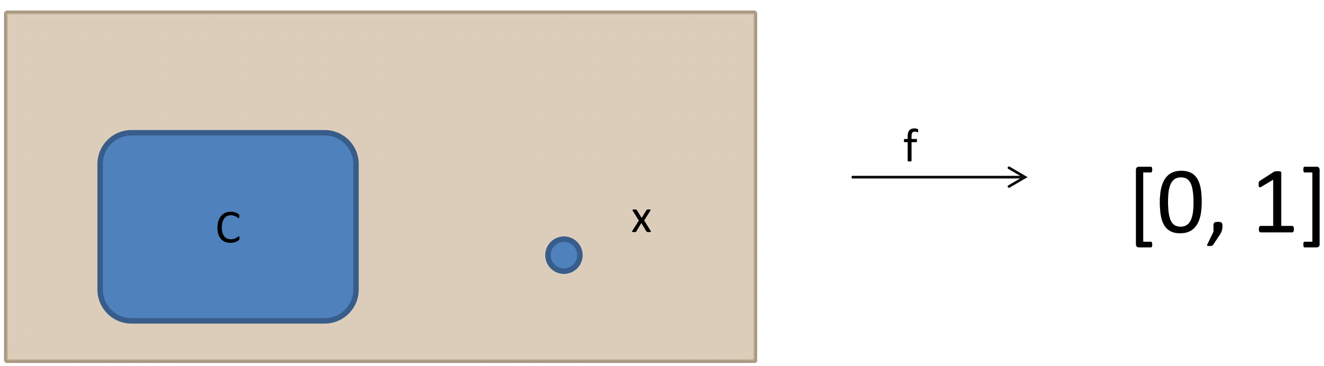
Normal

if any two disjoint closed subsets of *X* are separated by disjoint open sets.



Completely regular

If, given any point *x* and closed set *F* in *X* such that *x* does not belong to *F*, they are separated by a continuous function.



f(x) = 0 and f(y) = 1 for all y in C

Tychnoff space

If a space is completely regular and T1

T4-separation axiom
Tietze space

X fulfils T1 and is normal

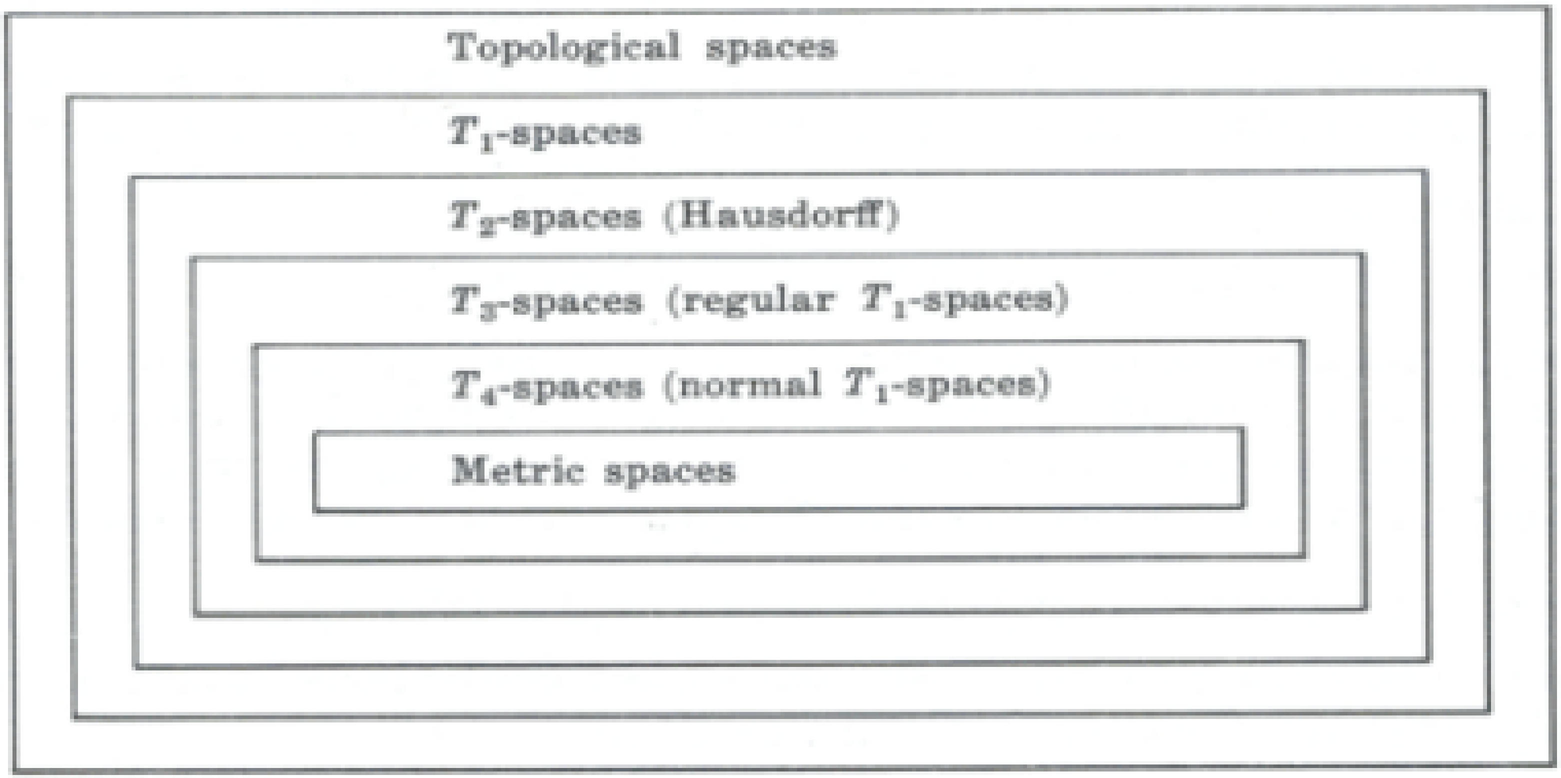

Topological spaces
T_1-spaces
T_2-spaces (Hausdorff)
T_3-spaces (regular T_1-spaces)
T_4-spaces (normal T_1-spaces)
Metric spaces

Fig. 4

- Reference Text:

Topology by K.D.Joshi