

PRESSURE OF AN IDEAL GAS – THE KINETIC GAS EQUATION

The pressure of a gas is due to the elastic collisions of the gas molecules on the walls of the container

Consider N molecules of a gas - each of mass 'm', enclosed within a cube of edge length 'l'.

Consider a single molecule with mass 'm' and velocity 'c'.

The velocity is resolved into three components cx, cy and cz along the x, y and z axes

$$c^2 = c_x^2 + c_y^2 + c_z^2$$



Consider the motion in the x-direction towards face A of the cube. The molecule will strike the face A with momentum mc_x .

it will rebound with a momentum $-mc_x$

Change in momentum = $mc_x - (-mc_x) = 2mc_x$

Velocity of the molecule is c_x m/s.

The distance travelled by the molecule in 1 second = c_x

The distance travelled by the molecule for colliding with face A = 2*l* Number of collisions on face A by the molecule in 1 second = $\frac{c_x}{2l}$



Change in momentum of a molecule per second = $2mc_x \times \frac{c_x}{2l} = \frac{mc_x^2}{l}$

Total change in momentum per second due to the impact on the two opposite faces along x-axis = $2mc_x^2$

 $\frac{2mc_x^{-2}}{l}$

The total change in momentum per molecule per second = $-\frac{1}{2}$

$$= \frac{2mc_{x}^{2}}{l} + \frac{2mc_{y}^{2}}{l} + \frac{2mc_{z}^{2}}{l}$$
$$= \frac{2mc^{2}}{l}$$

Let the individual velocities of molecules 1 to N be respectively c_1 , c_2 , c_3 to c_N .

The total change in momentum per second of N molecules = $\frac{2mc_1^2}{l} + \frac{2mc_2^2}{l} + \dots + \frac{2mc_N^2}{l}$



$$= \frac{2m}{l} \left(c_1^2 + c_2^2 + \dots c_N^2 \right)$$

$$= \frac{2mN}{l} \left(\frac{c_1^2 + c_2^2 + \dots + c_N^2}{N} \right)$$

$$= \frac{2mNu^2}{l}$$

 $u^{2} = \left(\frac{c_{1}^{2} + c_{2}^{2} + \dots + c_{N}^{2}}{N}\right) \text{ is Mean Square Velocity}$

u is known as the **Root Mean Square** (RMS) Velocity

The total change in momentum per second is equivalent to the force, f.

$$f = \frac{2mNu^2}{l}$$
$$P = \frac{force}{area} = \frac{f}{A}$$
$$P = \frac{force}{area} = \frac{2mNu^2}{l \times A}$$
$$P = \frac{2mNu^2}{l \times 6l^2} = \frac{mNu^2}{3l^3}$$

 $P = \frac{1}{mNu^2}$

 $A = 6l^2$

 $PV = \frac{1}{3}mNu^2$

 $V = l^3$

Kinetic Gas Equation