HEAT CAPACITY OF GASES

Suppose ‘q’ represents the quantity of heat required to raise the temperature of a system from T_1 to T_2.

$$q \propto (T_2 - T_1) \qquad q \propto \Delta T$$

$$q = C.\Delta T$$

$$\boxed{C = \frac{q}{\Delta T}}$$

The heat capacity of a system is defined as the quantity of heat required to raise the temperature of the system by 1 ^{0}C (or 1 K).

Specific Heat Capacity (s):

The quantity of heat required to raise the temperature of 1g of the substance by 1 ^{0}C (or 1 K).

$$s = \frac{C}{m} \qquad s = \frac{q}{m.\Delta T}$$

Molar Heat Capacity (C_m):

The quantity of heat required to raise the temperature of 1 mole of the substance by 1 ^{0}C (or 1 K).

$$C_m = \frac{C}{n} \qquad C_m = \frac{q}{n.\Delta T}$$

Two types of molar heat capacities are considered. Molar heat capacity at constant volume ($C_{V,m}$ or C_V) and Molar heat capacity at constant pressure ($C_{P,m}$ or C_P).

Molar heat capacity at constant volume ($C_{V,m}$ or C_V):

The quantity of heat required to raise the temperature of 1 mole of the substance by 1 ^{0}C (or 1 K) at constant volume. At constant volume, the heat supplied goes exclusively to increase the kinetic energy.

Kinetic energy of 1 mole of gas at temperature T K $= \tfrac{3}{2}RT$

Kinetic energy of 1 mole of gas at temperature (T+1) K $= \tfrac{3}{2}R(T+1)$

Increase in Kinetic energy for 1 K rise in temperature $= \tfrac{3}{2}R(T+1) - \tfrac{3}{2}RT = \tfrac{3}{2}R$

Since, the heat supplied goes exclusively to increase the kinetic energy, the heat capacity is given by:

$$C_V = \frac{q}{\Delta T}$$

$$C_V = \frac{\tfrac{3}{2}R}{(T+1)-T} \qquad C_V = \tfrac{3}{2}R$$

Molar heat capacity at constant pressure ($C_{P,m}$ or C_P):

The quantity of heat required to raise the temperature of 1 mole of the substance by 1 ^{0}C (or 1 K) at constant pressure. At constant pressure, the heat supplied goes to increase the kinetic energy as well as to perform the mechanical work of expansion.

Expansion work involved at constant pressure P by one mole of gas on heating through 1 K when its volume changes form V to $V+\Delta V = P\Delta V$

For an ideal gas, $PV = RT$ at temperature T K

At temperature T+1 K, the change in volume is $V+\Delta V$

Hence, $P(V+\Delta V) = R(T+1)$

$P\Delta V = R$

The heat supplied goes to increase the kinetic energy as well as to perform the mechanical work of expansion.

$$q = {}^{3}\!/_{2}\,R + P\Delta V$$

$$\therefore\quad q = {}^{3}\!/_{2}\,R + R = {}^{5}\!/_{2}\,R$$

$$C_P = \frac{q}{\Delta T}$$

$$C_P = \frac{{}^{5}\!/_{2}\,R}{(T+1)-T} \qquad\qquad C_P = {}^{5}\!/_{2}\,R$$

C_p & C_v Relations:

$$C_P - C_V = {}^{5}\!/_{2}R - {}^{3}\!/_{2}R$$

Or, $$C_P - C_V = R$$

The ratio of heat capacities is given by:

$$\frac{C_P}{C_V} = \gamma = \frac{{}^{5}\!/_{2}R}{{}^{3}\!/_{2}R} = \frac{5}{3}$$

Or, $$\frac{C_P}{C_V} = \gamma = 1.667$$

- The above conclusions are applicable only to ideal monoatomic gases, which possess only translational kinetic energy.
- Polyatomic molecules possess rotational and vibrational energies other than translational energy, hence their heat capacity also increases with increase in temperature.