

Statistical Quality Control

Three SQC Categories

Statistical quality control (SQC): the term used to describe the set of statistical tools used by quality professionals; SQC encompasses three broad categories of:

1. **Statistical process control (SPC)**
2. **Descriptive statistics include** the mean, standard deviation, and range
 - Involve inspecting the output from a process
 - Quality characteristics are measured and charted
 - Helps identify in-process variations
3. **Acceptance sampling** used to randomly inspect a batch of goods to determine acceptance/rejection
 - Does not help to catch in-process problems

Sources of Variation

- **Variation exists in all processes.**
- **Variation can be categorized as either:**
 - **Common or Random causes of variation, or**
 - Random causes that we cannot identify
 - Unavoidable, e.g. slight differences in process variables like diameter, weight, service time, temperature
 - **Assignable causes of variation**
 - Causes can be identified and eliminated: poor employee training, worn tool, machine needing repair

Descriptive Statistics

- **Descriptive Statistics include:**

- **The Mean**- measure of central tendency
- **The Range**- difference between largest/smallest observations in a set of data
- **Standard Deviation** measures the amount of data dispersion around mean
- **Distribution of Data shape**
 - Normal or bell shaped or
 - Skewed

$$\bar{\mathbf{X}} = \frac{\sum_{i=1}^{\mathbf{n}} \mathbf{x}_i}{\mathbf{n}}$$

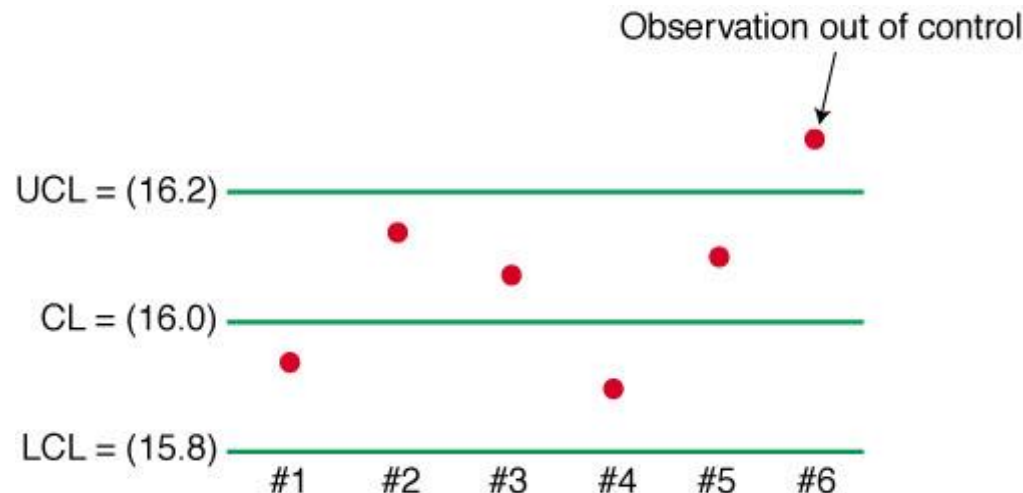
$$\sigma = \sqrt{\frac{\sum_{i=1}^{\mathbf{n}} (\mathbf{x}_i - \bar{\mathbf{X}})^2}{\mathbf{n} - 1}}$$

SPC Methods-Developing Control Charts

Control Charts show sample data plotted on a graph with CL, UCL, and LCL

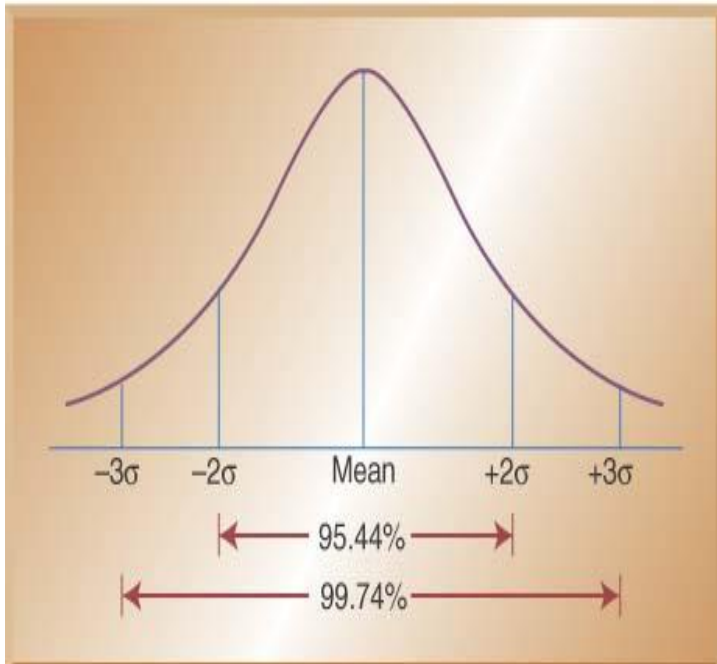
Control chart for variables are used to monitor characteristics that can be measured, e.g. length, weight, diameter, time

Control charts for attributes are used to monitor characteristics that have discrete values and can be counted, e.g. % defective, # of flaws in a shirt, etc.

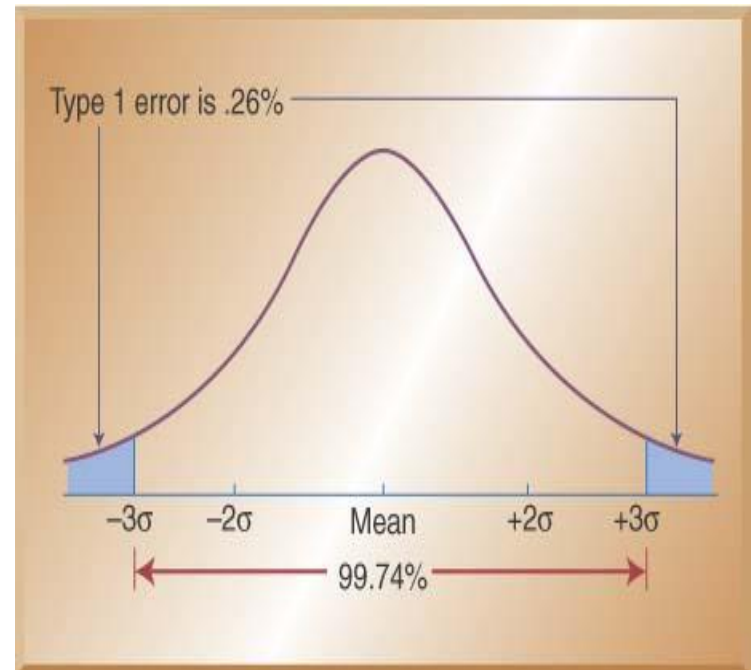


Setting Control Limits

- Percentage of values under normal curve

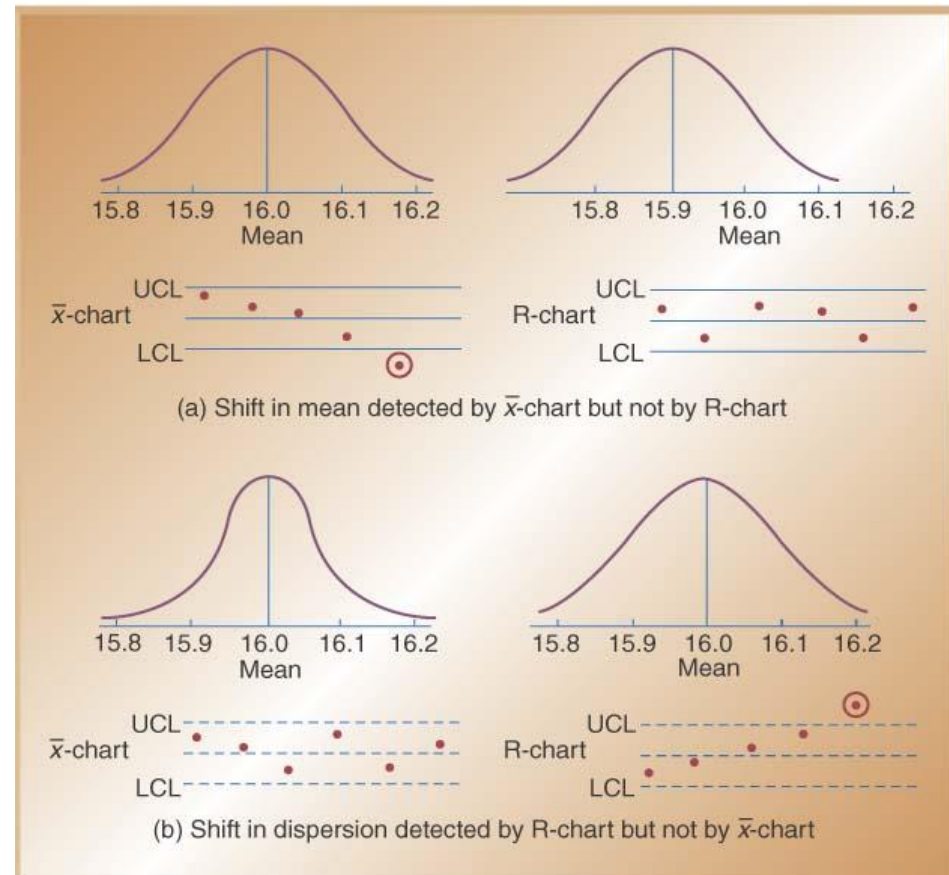


- Control limits balance risks like Type I error



Control Charts for Variables

- Use x-bar and R-bar charts together
- Used to monitor different variables
- X-bar & R-bar Charts reveal different problems
- Is statistical control on one chart, out of control on the other chart? OK?



Control Charts for Variables

- Use x-bar charts to monitor the changes in the mean of a process (central tendencies)
- Use R-bar charts to monitor the dispersion or variability of the process
- System can show acceptable central tendencies but unacceptable variability or
- System can show acceptable variability but unacceptable central tendencies

Constructing an X-bar Chart: A quality control inspector at the Cocoa Fizz soft drink company has taken **three samples with four** observations each of the volume of bottles filled. If the **standard deviation** of the bottling operation is **.2 ounces**, use the below data to develop control charts with limits of **3** standard deviations for the 16 oz. bottling operation.

| | Time 1 | Time 2 | Time 3 |
|----------------------|---------------|---------------|-------------|
| Observation 1 | 15.8 | 16.1 | 16.0 |
| Observation 2 | 16.0 | 16.0 | 15.9 |
| Observation 3 | 15.8 | 15.8 | 15.9 |
| Observation 4 | 15.9 | 15.9 | 15.8 |
| Sample means (X-bar) | 15.875 | 15.975 | 15.9 |
| Sample ranges (R) | 0.2 | 0.3 | 0.2 |

Center line and control limit formulas

$$\bar{\mathbf{x}} = \frac{\mathbf{x}_1 + \mathbf{x}_2 + \dots + \mathbf{x}_n}{k}, \quad \sigma_{\bar{\mathbf{x}}} = \frac{\sigma}{\sqrt{n}}$$

where (k) is the # of sample means and (n) is the # of observations w/in each sample

$$\text{UCL}_{\bar{\mathbf{x}}} = \bar{\mathbf{x}} + z\sigma_{\bar{\mathbf{x}}}$$

$$\text{LCL}_{\bar{\mathbf{x}}} = \bar{\mathbf{x}} - z\sigma_{\bar{\mathbf{x}}}$$

Solution and Control Chart (x-bar)

- **Center line (x-double bar):**

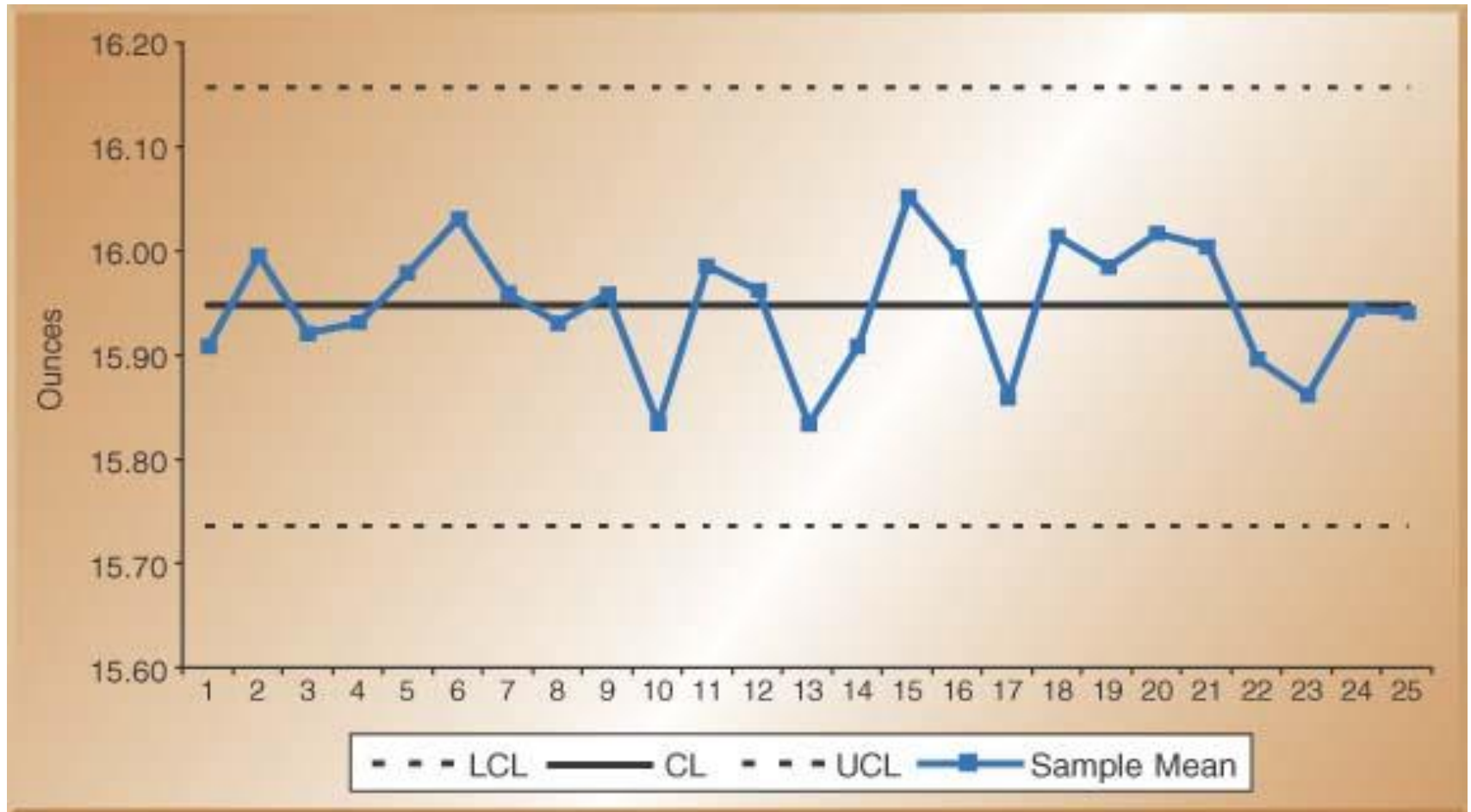
$$\bar{\bar{x}} = \frac{15.875 + 15.975 + 15.9}{3} = 15.92$$

- **Control limits for $\pm 3\sigma$ limits:**

$$UCL_{\bar{x}} = \bar{\bar{x}} + z\sigma_{\bar{x}} = 15.92 + 3\left(\frac{.2}{\sqrt{4}}\right) = 16.22$$

$$LCL_{\bar{x}} = \bar{\bar{x}} - z\sigma_{\bar{x}} = 15.92 - 3\left(\frac{.2}{\sqrt{4}}\right) = 15.62$$

X-Bar Control Chart



Control Chart for Range (R)

- Center Line and Control Limit formulas:

$$\bar{R} = \frac{0.2 + 0.3 + 0.2}{3} = .233$$

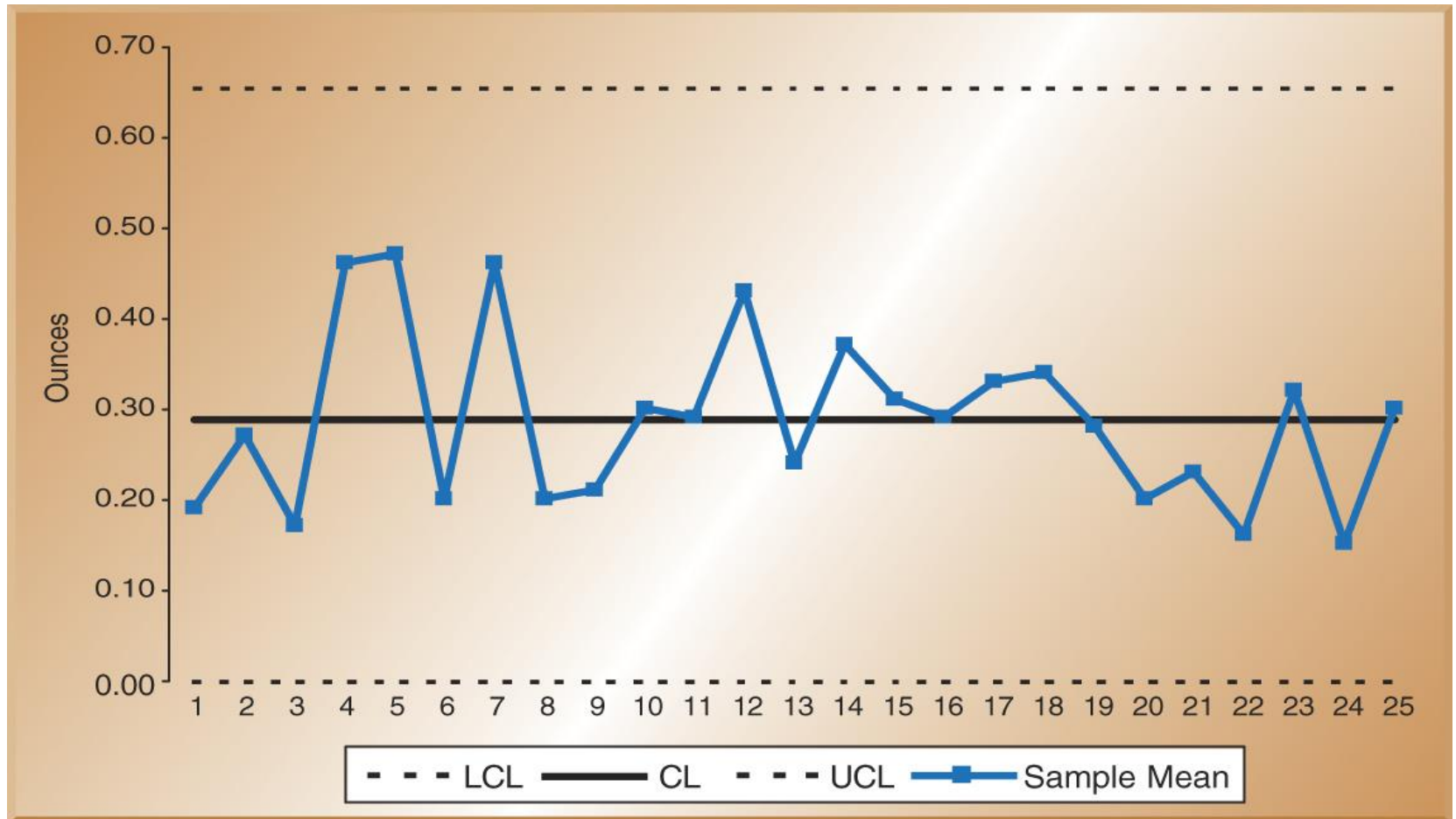
$$UCL_R = D_4 \bar{R} = 2.28(.233) = .53$$

$$LCL_R = D_3 \bar{R} = 0.0(.233) = 0.0$$

- Factors for three sigma control limits

| Sample Size (n) | Factor for x-Chart | Factors for R-Chart | |
|--------------------|--------------------|---------------------|------|
| | A2 | D3 | D4 |
| 2 | 1.88 | 0.00 | 3.27 |
| 3 | 1.02 | 0.00 | 2.57 |
| 4 | 0.73 | 0.00 | 2.28 |
| 5 | 0.58 | 0.00 | 2.11 |
| 6 | 0.48 | 0.00 | 2.00 |
| 7 | 0.42 | 0.08 | 1.92 |
| 8 | 0.37 | 0.14 | 1.86 |
| 9 | 0.34 | 0.18 | 1.82 |
| 10 | 0.31 | 0.22 | 1.78 |
| 11 | 0.29 | 0.26 | 1.74 |
| 12 | 0.27 | 0.28 | 1.72 |
| 13 | 0.25 | 0.31 | 1.69 |
| 14 | 0.24 | 0.33 | 1.67 |
| 15 | 0.22 | 0.35 | 1.65 |

R-Bar Control Chart



Second Method for the X-bar Chart Using R-bar and the A₂ Factor

- Use this method when sigma for the process distribution is not know
- Control limits solution:

$$\bar{\mathbf{R}} = \frac{\mathbf{0.2} + \mathbf{0.3} + \mathbf{0.2}}{\mathbf{3}} = \mathbf{.233}$$

$$\mathbf{UCL}_{\bar{\mathbf{x}}} = \bar{\mathbf{x}} + \mathbf{A}_2 \bar{\mathbf{R}} = \mathbf{15.92} + (\mathbf{0.73}) \cdot \mathbf{.233} = \mathbf{16.09}$$

$$\mathbf{LCL}_{\bar{\mathbf{x}}} = \bar{\mathbf{x}} - \mathbf{A}_2 \bar{\mathbf{R}} = \mathbf{15.92} - (\mathbf{0.73}) \cdot \mathbf{.233} = \mathbf{15.75}$$

P-Chart Example: A production manager for a tire company has inspected the number of defective tires in five random samples with 20 tires in each sample. The table below shows the number of defective tires in each sample of 20 tires. Calculate the control limits.

| Sample | Number of Defective Tires | Number of Tires in each Sample | Proportion Defective |
|--------|---------------------------|--------------------------------|----------------------|
| 1 | 3 | 20 | .15 |
| 2 | 2 | 20 | .10 |
| 3 | 1 | 20 | .05 |
| 4 | 2 | 20 | .10 |
| 5 | 2 | 20 | .05 |
| Total | 9 | 100 | .09 |

Solution:

$$CL = \bar{p} = \frac{\# \text{ Defectives}}{\text{Total Inspected}} = \frac{9}{100} = .09$$

$$\sigma_p = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = \sqrt{\frac{(.09)(.91)}{20}} = 0.064$$

$$UCL_p = \bar{p} + z(\sigma) = .09 + 3(.064) = .282$$

$$LCL_p = \bar{p} - z(\sigma) = .09 - 3(.064) = -.102 = 0$$

Control Charts for Attributes –P-Charts & C-Charts

Attributes are discrete events: yes/no or pass/fail

- Use P-Charts for quality characteristics that are discrete and involve yes/no or good/bad decisions
 - Number of leaking caulking tubes in a box of 48
 - Number of broken eggs in a carton
- Use C-Charts for discrete defects when there can be more than one defect per unit
 - Number of flaws or stains in a carpet sample cut from a production run
 - Number of complaints per customer at a hotel

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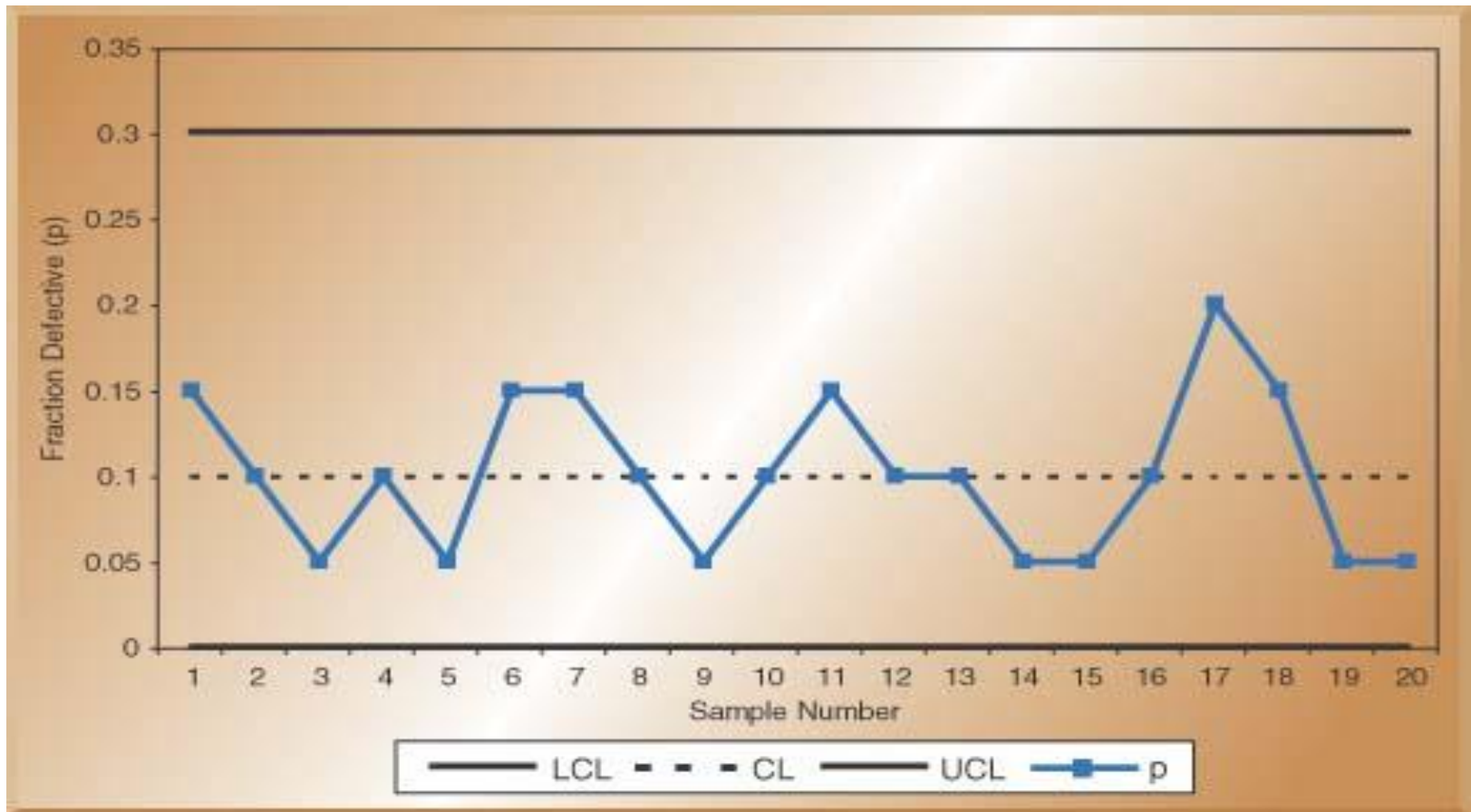
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P- Control Chart



C-Chart Example: The number of weekly **customer complaints** are monitored in a large hotel using a c-chart. Develop **three sigma control limits** using the data table below.

| Week | Number of Complaints |
|--------------|-----------------------------|
| 1 | 3 |
| 2 | 2 |
| 3 | 3 |
| 4 | 1 |
| 5 | 3 |
| 6 | 3 |
| 7 | 2 |
| 8 | 1 |
| 9 | 3 |
| 10 | 1 |
| Total | 22 |

Solution:

$$\mathbf{CL} = \frac{\text{\# complaints}}{\text{\# of samples}} = \frac{22}{10} = 2.2$$

$$\mathbf{UCL}_c = \bar{c} + z\sqrt{\bar{c}} = 2.2 + 3\sqrt{2.2} = 6.65$$

$$\mathbf{LCL}_c = \bar{c} - z\sqrt{\bar{c}} = 2.2 - 3\sqrt{2.2} = -2.25 = 0$$

C- Control Chart

