OPERATION RESEARCH

MODULE 1

INTRODUCTION TO OPERATION RESEARCH

Introduction

- Modern technological advance ➡ growth of scientific techniques
- Operations Research (O.R.) ➡ recent addition to scientific tools
- O.R. ➡ outlook to many conventional management problems
- Seeks the determination of best (optimum) course of action of a decision problem under the limiting factor of limited resources

What is O.R.?

• Operational Research can be considered as being the application of scientific method by **inter-disciplinary teams** to solve problems involving the control of organized **(man-machine systems)** so as to provide solutions which best serve the purposes of the organization as a whole.

NATURE OF O.R. : CHARACTERISTICS

- Inter-disciplinary team approach
- Systems approach
- Helpful in improving the quality of solution
- Scientific method
- Goal oriented optimum solution
- Use of models
- Require willing executives
- Reduces complexity by use of computers

SCOPE

Finance
Budgeting
and
investments

Purchasing
Procurement
and
Exploration

Production
Management

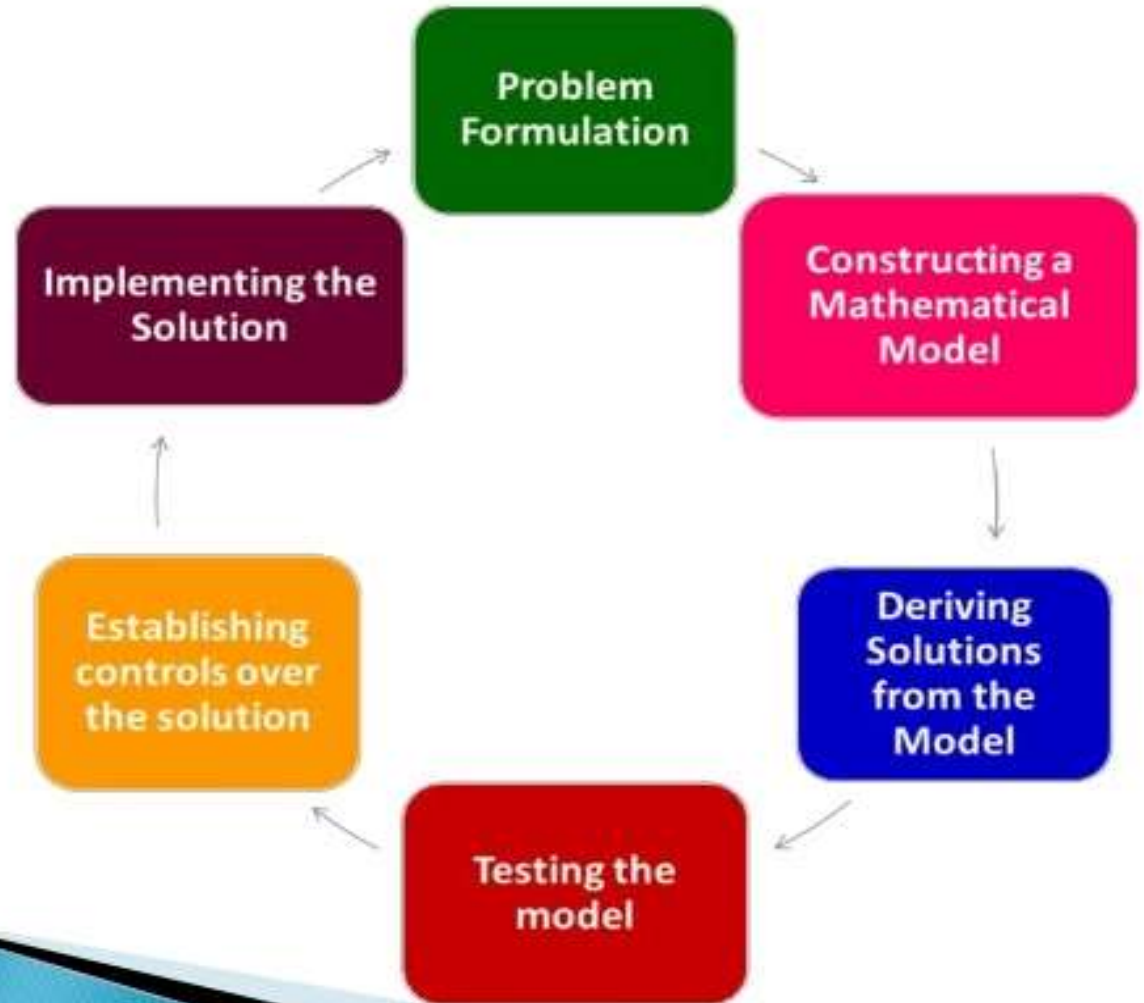
Marketing
Management

Personal
Management

PHASES OF O.R. STUDY

- **Judgment phase**
  - Determination of the problem
  - Establishment of the objectives and values
  - Determination of suitable measures of effectiveness
- **Research phase**
  - Observation and data collection
  - Formulation of hypothesis and models
  - Observation and experimentation to test the hypothesis
  -- Prediction of various results, generalization, consideration of alternative method
- **Action phase**
  - Implementation of the tested results of the model

METHODOLOGY



LIMITATIONS OF O.R.

- The inherent limitations concerning mathematical expressions
- High costs are involved in the use of O.R. techniques
- O.R. does not take into consideration the intangible factors
- O.R. is only a tool of analysis and not the complete decision-making process
- Other limitations
  - Bias
  - Inadequate objective functions
  - Internal resistance
  - Competence
  - Reliability of the prepared solution

MODULE II
LINEAR PROGRAMMING

LINEAR PROGRAMMING

- What is LP ?
  - The word linear means the relationship which can be represented by a straight line .i.e the relation is of the form

ax +by=c. In other words it is used to describe the relationship between two or more variables which are proportional to each other

The word "programming" is concerned with the optimal allocation of limited resources.

- Linear programming is a way to handle certain types of optimization problems
- Linear programming is a mathematical method for determining a way to achieve the best outcome

REQUIREMENTS

- There must be well defined objective function.
- There must be a constraint on the amount.
- There must be alternative course of action.
- The decision variables should be interrelated and non negative.
- The resource must be limited in supply.

ASSUMPTIONS

- Proportionality
- Additivity
- Continuity
- Certainity
- Finite Choices

ADVANTAGES OF L.P.

- It helps in attaining optimum use of productive factors.
- It improves the quality of the decisions.
- It provides better tools for meeting the changing conditions.
- It highlights the bottleneck in the production process.

LIMITATION OF L.P.

- For large problems the computational difficulties are enormous.
- It may yield fractional value answers to decision variables.
- It is applicable to only static situation.
- LP deals with the problems with single objective.

Types of Solutions to L.P. Problem

- Graphical Method
- Simplex Method

IMPORTANT DEFINITIONS IN L.P.

- Solution:

  A set of variables [X1,X2,...,Xn+m] is called a solution to L.P. Problem if it satisfies its constraints.

- Feasible Solution:

  A set of variables [X1,X2,...,Xn+m] is called a feasible solution to L.P. Problem if it satisfies its constraints as well as non-negativity restrictions.

- Optimal Feasible Solution:

  The basic feasible solution that optimises the objective function.

- Unbounded Solution:

  If the value of the objective function can be increased or decreased indefinitely, the solution is called an unbounded solution.

VARIABLES USED IN L.P.

- Slack Variable
- Surplus Variable
- Artificial Variable

Non-negative variables, Subtracted from the L.H.S of the constraints to change the inequalities to equalities. Added when the inequalities are of the type (>=). Also called as “negative slack”.

- Slack Variables

Non-negative variables, added to the L.H.S of the constraints to change the inequalities to equalities. Added when the inequalities are of the type (<=).

- Surplus Variables

In some L.P problems slack variables cannot provide a solution. These problems are of the types (>=) or (=) . Artificial variables are introduced in these problems to provide a solution.

Artificial variables are fictitious and have no physical meaning.

- Artificial Variables

DUALITY :

- For every L.P. problem there is a related unique L.P. problem involving same data which also describes the original problem.
- The primal programme is rewritten by transposing the rows and columns of the algebraic statement of the problem.
- The variables of the dual programme are known as “Dual variables or Shadow prices” of the various resources.
- The optimal solution of the dual problem gives complete information about the optimal solution of the primal problem and vice versa.

MODULE III
TRANSPORTATION PROBLEMS

TERMINOLOGY USED IN TRANSPORTATIONAL MODEL

Feasible solution: Non negative values of xij where i=1, 2…….m and j=1, 2,…n which satisfy the constraints of supply and demand is called feasible solution.

Basic feasible solution: If the no of positive allocations are (m+n-1).

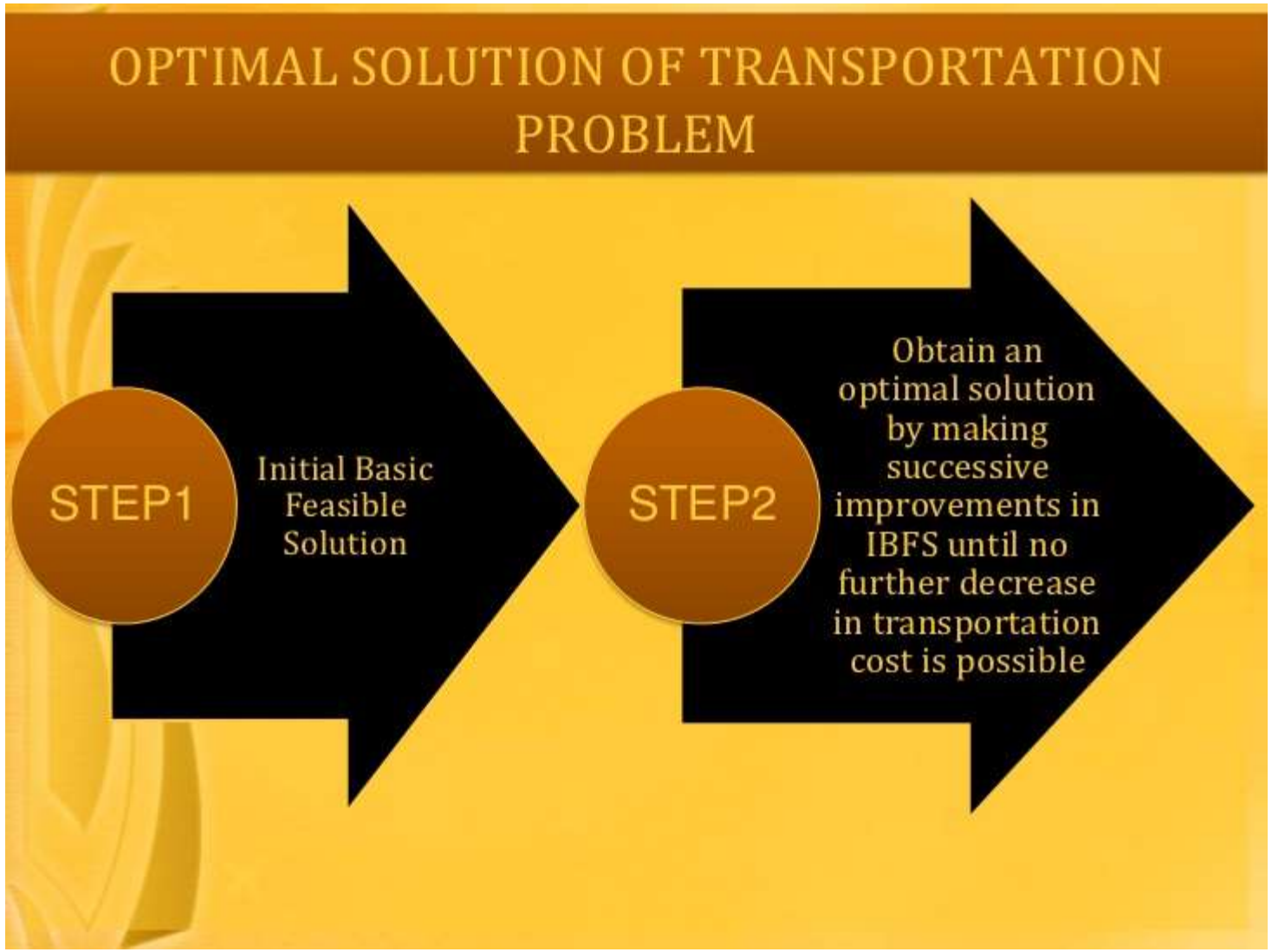
Optimal solution: A feasible solution is said to be optimal solution if it minimizes the total transportation cost.

Balanced transportation problem: A transportation problem in which the total supply from all sources is equal to the total demand in all the destinations.

Unbalanced transportation problem: Problems which are not balanced are called unbalanced.

Matrix terminology: In the matrix, the squares are called cells and form columns vertically and rows horizontally.

Degenerate basic feasible solution: If the no. of allocation in basic feasible solutions is less than (m+n-1).


OPTIMAL SOLUTION OF TRANSPORTATION PROBLEM
STEP1
Initial Basic Feasible Solution
STEP2
Obtain an optimal solution by making successive improvements in IBFS until no further decrease in transportation cost is possible

INITIAL BASIC FEASIBLE SOLUTION

Methods Available

NORTH WEST CORNER METHOD(NWCM)	LOWEST COST ENTRY METHOD(LCEM)	VOGEL'S APPROXIMATION METHOD(VAM)

UNBALANCED TRANSPORTATION PROBLEM

Demand>Supply

Add dummy column in matrix with zero cost

Supply>Demand

Add dummy row in matrix with zero cost

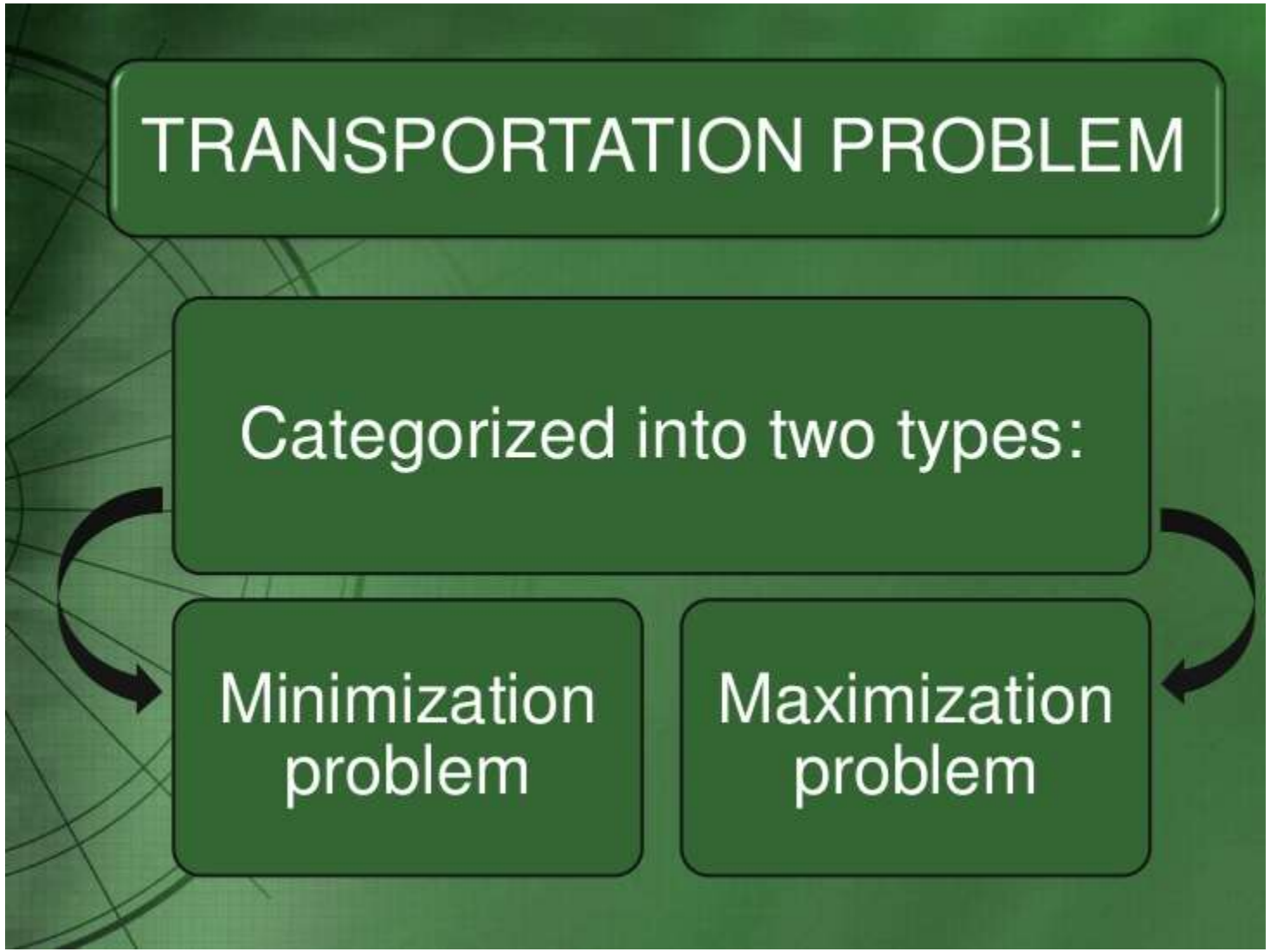
METHOD FOR OPTIMAL SOLUTION

Stepping stone method

Modified Distribution Method(MODI)

LEAST COST ENTRY METHOD

This method takes into consideration the lowest cost and therefore takes less time to solve the problem


TRANSPORTATION PROBLEM
Categorized into two types:
Minimization problem
Maximization problem

MINIMIZATION PROBLEM

In this transportation cost is given which is to be minimized.

MAXIMIZATION PROBLEM

In this profit is given which is to be maximized.

To solve this problem we convert the problem into minimization.

Conversion is done by selecting the largest element from Profit Pay off matrix and then subtracting all elements from largest element including itself.

Reduced matrix obtain becomes minimization case and then same steps are taken to solve it as is done in minimization problem

VOGEL'S APPROXIMATION METHOD

- BASIS OF ALLOCATION IS UNIT COST PENALTY
- THE SUBSEQUENT ALLOCATIONS IN CELLS ARE DONE KEEPING IN VIEW THE HIGHEST UNIT COST
- IBFS OBATINED BY THIS METHOD IS EITHER OPTIMAL OR VERY NEAR TO OPTIMAL SOLUTION
- SO AMOUNT OF TIME REQUIRED TO CALCULATE THE OPTIMUM SOLUTION IS REDUCED

MODULE IV
DECISION THEORY

Decision Theory

Decision Theory **represents a general approach to decision making which is suitable for a wide range of management decisions, including:**

- Capacity planning
- product and service design
- location planning
- equipment selection
- Product –mix
- Credit policies

Problem Formulation

- A decision problem is characterized by decision alternatives, states of nature, and resulting payoffs.
- The decision alternatives are the different possible strategies the decision maker can employ.
- The states of nature refer to future events, not under the control of the decision maker, which will ultimately affect decision results.
- States of nature should be defined so that they are mutually exclusive and contain all possible future events that could affect the results of all potential decisions.

Payoff Tables

- The consequence resulting from a specific combination of a decision alternative and a state of nature is a payoff.
- A table showing payoffs for all combinations of decision alternatives and states of nature is a payoff table.
- Payoffs can be expressed in terms of profit, cost, time, distance or any other appropriate measure.

Fundamentals of decision theory

Decision alternatives	States of nature	Payoff
Courses of action or strategies	An occurrence over which decision maker has no control	Quantitative measure of the outcome

Risk vs. Uncertainty

- Risk
  - Must make a decision for which the outcome is not known with certainty
  - Can list all possible outcomes & assign probabilities to the outcomes
- Uncertainty
  - Cannot list all possible outcomes
  - Cannot assign probabilities to the outcomes
- Certainty

  -is an environment in which future outcomes or state of nature are known.
- Eg: Investment in Bank FD, there is CERTAINTY regarding FUTURE PAYMENTS on maturity
- Investment in shares is risky
- Investment in shares FETCHING returns higher than FD in another 2 years, is uncertain

Criteria of decision making under uncertainity

- Optimism(Maximax or Minimin)
- Pessimism(Maximin or Minimax)
- Equal probabilities(Laplace)
- Coefficient of optimism(Hurwicz)
- Regret(Salvage)

Decision tree

- Decision tree is a network which exhibits graphically the relationship between the different parts of the complex decision process.
- It is a graphical model of each combination of various acts and states of nature along with their payoffs, probability distribution
- It is extremely useful in multistage situations which involve a number of decisions ,each depending on the preceding one.
- A decision tree analysis involves the construction of a diagram that shows , at a glance, when decisions are expected to be made- in what sequence, their possible outcomes, & corresponding payoffs.

- A DT consists of nodes, branches, probability estimates and pay-offs
- Three types of “nodes”
  - Decision nodes - represented by squares (□) It represents a point of action where a decision maker must select one alternative course of action among the available
  - Chance nodes - represented by circles (O) It indicates a point of time where the decision maker will discover the response to his decision
  - Terminal nodes - represented by triangles (optional)
- Solving the tree involves pruning all but the best decisions at decision nodes, and finding expected values of all possible states of nature at chance nodes
- Create the tree from left to right
- Solve the tree from right to left

Decision making under risk

- ☑ Each possible state of nature has an assumed probability p_i
- ☑ States of nature are mutually exclusive
- ☑ Probabilities must sum to 1
- ☑ Determine the expected monetary value (EMV) for each alternative

Expected monetary value $= \sum_{i=1}^{n} p_i X_i$

Where X_i is the i^{th} outcome of a decision, p_i is the probability of the i^{th} outcome, and n is the total number of possible outcomes

Expected Opportunity Loss (EOL)

- It is the opposite of EMV
- EOL is defined as the difference between the highest profit or pay-off and the actual profit due to choosing a particular course of action in a particular state of nature
- The conditional opportunity loss (EOL) for a particular course of action is determined by taking the difference between the payoff value of the most favourable course of action and some other course of action.

MODULE V
NETWORK ANALYSIS

Introduction

- Network analysis is the synthesis of two most useful techniques of project management i.e P.E.R.T and C.P.M.
- PERT was developed as a management tool for coordination and early completion of Polaris Ballistic Missile Project in U.S.A, resulting in reduction of 30% time in project execution.
- A contemporary of PERT is CPM and was developed in connection with maintenance and construction work.
- Both the techniques use, PERT and CPM use ‘network’ and ‘critical path’ as the basic tool of project management and both are very helpful in completing a project on schedule by coordinating different jobs involved in this completion.

What is a network?

A graphical presentation or an arrow diagram

presented to the management

in respect of a project

which consists of all details regarding consumption of **time** and **cost** not only for each activity but also for the whole project

so that management can manipulate the resources and cost can be controlled in a more effective manner.

Steps Involved In Process Of Network Analysis

- Identify the jobs, events or activities.
- Arrange the jobs in logical sequence.

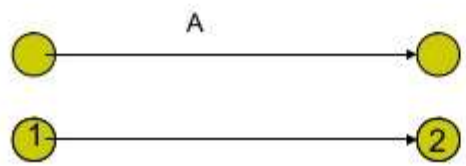
Objectives of Network Analysis

Network Analysis is a successful technique frequently used to plan, monitor and control the projects involving thousands of activities.

1. To minimize project cost.
2. To minimize the project time.
3. To ensure optimum utilization of human and other resources.
4. To ensure minimum conflicts and unnecessary delays.

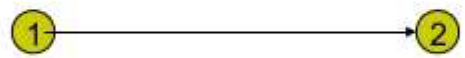
Basic key concepts in network analysis

- **Activity**- An activity refers to the physically identifiable part of the project which consumes time and cost. An activity is represented by an arrow. The activity may have a predecessor activity and a successor activity.
- E.g.-



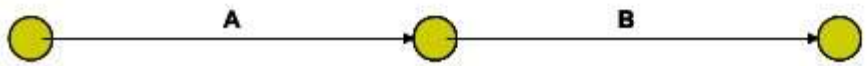
What is an **event**?

An event represents the start and the end point of an activity. It is normally represented by a circle called nodes.



EVENT EVENT

1 represents tail event or starting event. 2 represents end event or head event.



A is a preceding activity for B. B is a succeeding activity for A.

Critical Path Method (CPM)

- Critical Path Analysis is a special application of network analysis. It uses network analysis for scheduling production, construction projects as well as research and development activities.

  Besides this, it is used in certain other situations that require estimates of time and performance.

Dummy Activity

An activity which doesn’t consume any time and productive time and cost. It should be represented by dotted line but not by any alphabet.

It should be introduced to maintain preceding and succeeding relation.